Functions

A. The Concept of a Function

1. The concept of a function is both a new way of thinking and a new way of notating.

2. Let's look at a linear relationship the "old" way and with function notation.

   a. Given: $y = 2x + 1$

   b. We say that x is the *independent variable* and y is the *dependent variable*.

      Don't we choose values of x and then see what we get back get back for y?

   c. Another way to say this is: "The value of y *depends upon* the value of x."

      Yet another way to say this is: "The value of y is a function of our choice for x."

      When this statement is written using mathematical symbols, it becomes:

$$y = f(x)$$

Read, "y equals f of x."

"y is a function of x"

   d. Notice a most important fact: *y and f(x) may be used interchangeably;* they both mean *"what you get back after you have chosen a particular x value."*

   e. Let's look at both the old way and the new way of notating some values of x and y which satisfy our equation of $y = 2x + 1$.

OLD WAY: $Y = 2x + 1$	FUNCTIONS: $f(x) = 2x + 1$
When $x = 6$, $y = 13$	$f(6) = 13$
When $x = 0$, $y = 1$	$f(0) = 1$
When $x = -3$, $y = -5$	$f(-3) = -5$

When I put –3 into the function names f I get back a –5.

B. Function Notation

Question: "If $f(x) = 2x + 1$, what does this function do to anything (any x) you put into it?"

Answer: "It multiplies by two, and then adds one."

You may think of a function as a machine that sits in the corner and "does something" to each "thing" you put into it.

Question: "If $f(x) = x^2 - 2x$, what does this function *do* to each 'thing' you put into it?"

Answer: "It squares it, then subtracts two times the thing you put into it."

Then , f(5) means that 5 is what is going into the machine, and the machine will square the five, and then subtract two times the five.

$$\begin{aligned} f(5) &= (5)^2 - 2(5) \\ &= 25 - 10 \\ &= 15 \end{aligned}$$

This is very confusing at first, but keep working on it until it makes sense.

Note: f(5) tells us that five is what is going into the machine.

Since $f(5) = 15$, then f(5) *means* "what we get back out of the machine when we put a five into it."

Let's change functions now and make up a new one.

Let $g(x) = x^2 + 3$

Question: "What does this function do to anything you put into it?"

Answer: "It first squares it, then adds three."

1. So, $g(4) = 4^2 + 3 = 16 + 3 = 19$

   (Note that 4 is what went into the function, 19 is what we got back out of the function.)

2. $g(7) =$

3. $g(-1) =$

4. $g(x + 1) =$

Answers to Problems:

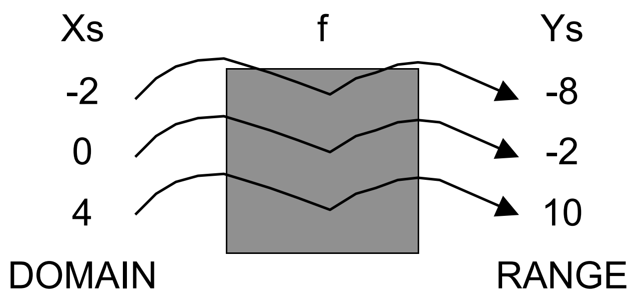
2. $g(7) = 7^2 + 3 = 49 + 3 = 52$

3. $g(-1) = (-1)^2 + 3 = 1 + 3 = 4$

4. $g(x + 1) = (x + 1)^2 + 3 = x^2 + 2x + 1 + 3 = x^2 + 2x + 4$

C. The Machine Picture of a Function

One way to get a better grasp of the concept of a function is to picture the function as a machine:



This is a picture of the function $f(x) = 3x - 2$. With -2, 0, and 4 being inputted into the function.

Note that the function is doing the same thing to each x, namely, multiplying by three, and then subtracting two.

The set of all *allowable* x (or input) numbers is called the DOMAIN.

The set of all *resulting* y (or output) numbers is called RANGE.

In this example, we would say the DOMAIN is { -2, 0, 4 } and that the RANGE is { -8, -2, 10 }.

D. The Formal Definition of a Function

> "A function is a relationship which assigns to each input (or domain) value, a unique output (or range) value."

Be sure to read this definition several times. You may also want to read the definition given in your textbook. The definition of a function may be stated in several ways. One definition may make more sense to you than another.

We will examine this definition from several perspectives in order to better understand it.

E. A Function as a Set of Ordered Pairs

Using the definition of a function, is each of the following sets of ordered pairs a function? (Answer "yes" or "no")

1. {(1,2), (3,5), (-2,8)} ____________

2. {(5,1), (7,1), (10, 1), (-8,2)} ______

3. {(2,6), (3,5), (4,1), (3,7)} _________

(answers and discussion on the next page)

Answers and Discussion:

1. Yes, this is a function. It does not violate the definition for a function. For instance, *every time* we let x = 1, we got only one unique value back for y, namely, a 2.

   The same is true for every other choice for x. For instance, *every time* we used an x value of 3, we got back a 5.

2. Yes, this is also a function. Reread the formal definition of a function in order to see how this *does not* violate that definition.

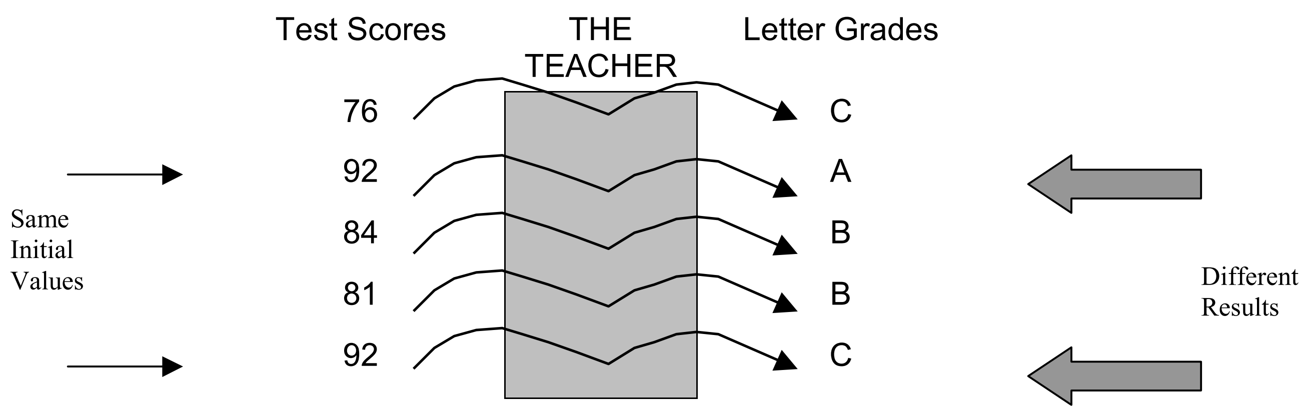
3. No, not a function. When we let x = 3 *on one occasion* we got back a 5. However, *the next time* we let x = 3, we got back a value of 7.

F. Functions Represent Consistent Relationships

WE STUDY FUNCTIONS BECAUSE THEY REPRESENT CONSISITENT RELATIONSHIPS, the kind of relationships we work with in science and in everyday life.

Alternate definition of a function: "A function is a consistent relationship such that **when an initial value is repeated (the x), we get the same result** (the y, or f(x))."

To illustrate, look at the machine picture of the function below:



What's wrong with this picture? **Do you see that when the input value of 92 was repeated, this teacher gave a different grade than the first time?** The relationship pictured here is inconsistent and is not a function.

Another example: *Everytime* we heat water to 100 degrees Centigrade (at normal atmospheric pressure) we expect it to boil. When we repeat the initial conditions, we expect the same results!

G. Conclusion

I hope this has helped give you a better idea of what a function is, how to use function notation, and to answer, in some measure, that nagging question "Why do we do this?"

Pick up the handout on Domain and Range if you wold like more detailed help on those two concepts.