Mini-Project 1: The Library of Functions

In your previous math class, you learned to graph equations containing two variables by finding and plotting points. In this class, we continue this skill by learning to graph functions that are often used in algebra. Collectively, we call these functions the library of functions.

To graph a function from the library of functions, follow these steps:
1. Assume a value for \( x \), then solve for the value of \( y \). (Remember that \( f(x) \) is the same thing as \( y \).)
2. Express these values of \( x \) and \( y \) as an ordered pair.
3. Repeat Steps 1 and 2 until you have enough points.
4. Determine the scale of the \( x \)-axis and the \( y \)-axis.
5. Plot the ordered pairs, then connect the points to graph the equation.

We will practice each of these steps, starting with Steps 1, 2 and 3.

Steps 1, 2, and 3

In class, we learned the values of \( x \) we should use to graph each of the functions in the library of functions. Find those values in your lecture notes, then answer these questions:

1. What values of \( x \) should you use if you are graphing the Constant Function? (Circle One.)
   
   [A] -2, -1, 0, 1, and 2  [B] 0, 1, 4, 9, and 16  [C] -8, -1, 0, 1, and 8  [D] -4, -2, -1, \(-\frac{1}{2}\), \(-\frac{1}{4}\), \(\frac{1}{2}\), 1, 2 and 4

2. What values of \( x \) should you use if you are graphing the Square Root Function? (Circle One.)
   
   [A] -2, -1, 0, 1, and 2  [B] 0, 1, 4, 9, and 16  [C] -8, -1, 0, 1, and 8  [D] -4, -2, -1, \(-\frac{1}{2}\), \(-\frac{1}{4}\), \(\frac{1}{2}\), 1, 2 and 4

3. What values of \( x \) should you use if you are graphing the Cube Function? (Circle One.)
   
   [A] -2, -1, 0, 1, and 2  [B] 0, 1, 4, 9, and 16  [C] -8, -1, 0, 1, and 8  [D] -4, -2, -1, \(-\frac{1}{2}\), \(-\frac{1}{4}\), \(\frac{1}{2}\), 1, 2 and 4

4. What values of \( x \) should you use if you are graphing the Reciprocal Function? (Circle One.)
   
   [A] -2, -1, 0, 1, and 2  [B] 0, 1, 4, 9, and 16  [C] -8, -1, 0, 1, and 8  [D] -4, -2, -1, \(-\frac{1}{2}\), \(-\frac{1}{4}\), \(\frac{1}{2}\), 1, 2 and 4
Once the values of $x$ have been determined, plug each one in, one at a time, to find the corresponding values of $y$. Then express each $x$- and $y$-value as an ordered pair.

For each function listed below, use the $x$-values to find the corresponding $y$-values, then express them as an ordered pair.

5. The Square Function
   \[ f(x) = x^2 \]
   
   \[ \begin{array}{|c|c|c|} 
   \hline
   x & y & (x,y) \\
   \hline
   -2 & 4 & \\
   -1 & 1 & \\
   0 & 0 & \\
   1 & 1 & \\
   2 & 4 & \\
   \hline
   \end{array} \]

6. The Identity Function
   \[ f(x) = x \]
   
   \[ \begin{array}{|c|c|c|} 
   \hline
   x & y & (x,y) \\
   \hline
   -2 & -2 & (0,0) \\
   -1 & -1 & \\
   0 & 0 & \\
   1 & 1 & \\
   2 & 2 & \\
   \hline
   \end{array} \]

7. The Cube Root Function
   \[ f(x) = \sqrt[3]{x} \]
   
   \[ \begin{array}{|c|c|c|} 
   \hline
   x & y & (x,y) \\
   \hline
   -8 & -2 & \\
   -1 & -1 & \\
   0 & 0 & \\
   1 & 1 & \\
   8 & 2 & \\
   \hline
   \end{array} \]

For the reciprocal function, each $x$-value is the reciprocal of its corresponding $y$-value and vice versa. For example, if $x = \frac{1}{4}$, then $y = 4$ or simply 4. If a value is negative, then its reciprocal is negative. For example, the reciprocal of $-2$ is $-\frac{1}{2}$.

8. The Reciprocal Function
   \[ f(x) = \frac{1}{x} \]
   
   \[ \begin{array}{|c|c|c|} 
   \hline
   x & y & (x,y) \\
   \hline
   -4 & -\frac{1}{4} & \\
   -2 & -\frac{1}{2} & \\
   -1 & -1 & \\
   -\frac{1}{2} & -2 & \\
   -\frac{1}{4} & -4 & \\
   \frac{1}{4} & 4 & \\
   \frac{1}{2} & 2 & \\
   1 & 1 & \\
   2 & \frac{1}{2} & \\
   4 & \frac{1}{4} & \\
   \hline
   \end{array} \]
Step 4
To determine the scale of the $x$-axis and the $y$-axis, identify the smallest value and the largest value in your table of points to be plotted. Then use those numbers to pick a starting number and an ending number to put on your $x$-axis and $y$-axis. These numbers should have the following properties:

- The starting number should be less than or equal to the smallest number in your table.
- The ending number should be greater than or equal to the largest number in your table.
- The starting and ending numbers should be a multiple of 5; for example you can use starting and ending numbers from this collection of numbers: {...-15, -10, -5, 0, 5, 10, 15, ...} (Exception: the reciprocal function will use a special scale from -4 to 4).
- The distance from the starting number to the ending number should be as small as possible (while still meeting all of the other criteria above).

Once you’ve determined your starting and ending numbers, put them on the $x$-axis and the $y$-axis, as well as all of the other multiples of 5 that are in between them (if any). (Exception: The number 0 is never labeled; it is understood that the point where the $x$-axis and the $y$-axis cross has a value of 0.)

For example, if your smallest value is $-13$ and your largest value is $2$, your starting number would be $-15$ and your ending number would be $5$. You would write $-15, -10, -5, 5$ on each axis. In this case, the origin would not be in the center of the graph.
9. For the table below, identify the starting and ending numbers, and the numbers that should be labeled on the x-axis and y-axis. (Circle One.)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ \text{A} \] Starting number: 0   Ending number: 16   Label: \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}

\[ \text{B} \] Starting number: -20  Ending number: 20  Label: \{-20,-10,10,20\}

\[ \text{C} \] Starting number: 0   Ending number: 15   Label: \{5,10,15\}

\[ \text{D} \] Starting number: 0   Ending number: 20   Label: \{5,10,15,20\}

10. For the table below, identify the starting and ending numbers, and the numbers that should be labeled on the x-axis and y-axis. (Circle One.)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-8</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

\[ \text{A} \] Starting number: -5  Ending number: 5  Label: \{-5,5\}

\[ \text{B} \] Starting number: -10  Ending number: 10  Label: \{-10,-5,5,10\}

\[ \text{C} \] Starting number: -2  Ending number: 8  Label: \{-2,3,8\}

\[ \text{D} \] Starting number: -2  Ending number: 2  Label: \{-2,-1,1,2\}
When making the $x$-axis and the $y$-axis for the reciprocal function, you must be careful because fractions are involved. To learn how to create a number line with fractions, follow the steps below.

First, draw a number line like you would normally see in a textbook: all integers with zero in the middle.

Next, divide each value by the largest denominator of all the fractions you need to plot. For the reciprocal function, that value is 4. This will ensure your fractions are in the right place, in the right order.

Finally, reduce each fraction to lowest terms.

Note that for the reciprocal function, not all fraction values must be labeled. The only ones you will be using are shown below.

Note also that there are four spaces between 0 and 1. So there should be four spaces between any adjacent integers. Use this idea to find where to place the numbers $-4, -2, 2,$ and $4$. 

NOTE: You do not have to write on this page. Just read and learn.

Step 5
It is a good idea to memorize the shapes of all eight functions in the library of functions. This will help you to graph them correctly when it comes time to plot your points and connect them. Consider the following:

The Square Function  The Constant Function  The Square Root Function

The Identity Function  The Cube Function  The Cube Root Function

The Reciprocal Function  The Absolute Value Function

The Constant, Identity, and Absolute Value functions should be drawn using a straightedge. The reciprocal function has two asymptotes: \( y = 0 \) (the \( x \)-axis) and \( x = 0 \) (the \( y \)-axis). Graph them! (Recall that asymptotes are dashed lines that the curve forever approaches but never reaches.)
You are now ready to graph each function in the library of functions. There are eight tables and eight graphs. The first thing you should do is figure out which function goes with which graph based on the part of the graph shown and the information provided. Once you’ve done that, create all eight graphs. Fill in any missing information, including the function name, the function equation, and the table of points. Be sure to use a straightedge where appropriate!

11. Function Name: 

**The Identity Function**

Function Equation:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. Function Name:

Function Equation:

$f(x) = 4$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
13. Function Name: 

Function Equation: 
\[ f(x) = x^2 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

14. Function Name: 

Function Equation: 

See the diagram on Page 6. Graph both asymptotes using dashed lines!

For this graph only: Next to each point you plot, label its coordinates with an ordered pair.
15. Function Name:

Function Equation:

\[
\frac{g_{1876}}{g_{1877}} \frac{g_{4666}}{g_{1876} \cdot g_{1877} \cdot g_{4667}}
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. Function Name:

The Absolute Value Function

Function Equation:

\[
\frac{g_{3398}}{g_{1876} \cdot g_{1877} \cdot g_{4666}}
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2)</td>
<td>2</td>
<td>(2, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### 17. Function Name:

**Function Equation:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-8$</td>
<td>$-2$</td>
<td></td>
</tr>
</tbody>
</table>

### 18. Function Name:

**Function Equation:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(2, 8)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Checklist for a Perfect Graph
For each of the graphs you created, now go back and make sure you’ve satisfied the following requirements:

- The x-axis is drawn and labeled with an “x”.
- The y-axis is drawn and labeled with a “y”.
- The values on the x- and y-axis are stated.
- The values on the x-axis and y-axis are consistent. For example, ...
  - If there are four spaces between 0 and 1, then there should be four spaces between 1 and 2, and so on.
  - If there are five spaces between 0 and 5, then there should be five spaces between 5 and 10, and so on.
- Correct points are plotted, and they are connected with the correct shape.
- The graph continues to the edge of the grid; arrows indicate that the graph continues beyond it.
- Straight lines and line segments are drawn with a straightedge.
- The graphs all look like those shown on Page 6.