Intermediate Algebra

§ 5.1/5.2 Exponents & Scientific Notation

Please turn to this section of the lecture notes.

Review of Exponent Rules

Memorize (or re-memorize) these rules.

Proof that \((x + y)^2 \neq x^2 + y^2\)

If you're unsure if you remembered a rule correctly, you can always plug in values for the variables. For example, if you thought

\[(x + y)^2 = x^2 + y^2\]

you could prove your error by plugging in.

Let's let \(x = 3\) and \(y = 4\). Then

\[(x + y)^2 = (3 + 4)^2 = 7^2 \neq 9 + 16 = 25\]
§5.1/5.2 Exponents & Scientific Notation

Please turn to this section of the lecture notes.

Review of Exponent Rules

Memorize (or re-memorize) these rules.

Proof that \((x+y)^2 \neq x^2 + y^2\):

If you're unsure if you remembered a rule correctly, you can always plug in values for the variables. For example, if you thought

\[
(x+y)^2 = x^2 + y^2
\]

you could prove your error by plugging in.

Let's let \(x = 3\) and \(y = 4\). Then

\[
(x+y)^2 \neq x^2 + y^2
\]

\[
(3+4)^2 \neq 3^2 + 4^2
\]

\[
7^2 \neq 9 + 16
\]

\[
49 \neq 25
\]
Evaluate both expressions:

\[ (-2)^0 = 1 \]
\[ (-2)^0 = 1 \]

Evaluate the expression:

\[ (-6)^{-2} = \frac{1}{(-6)^2} = \frac{1}{36} \]
Another helpful rule:

\[
\left( \frac{a}{b} \right)^{-n} = \left( \frac{b}{a} \right)^n
\]

Adding MnGP 20 (in §5.2)

Simplify:

\[
\left( \frac{4a^2 b^5}{8a^2 b^{-7}} \right)^{-3}
\]

\[
\left( \frac{8a^2 b^{-7}}{4a^2 b^5} \right)^3
\]

Which rule should we use next?

\[
\left( \frac{8}{4} \cdot \frac{a^2}{a^{-2}} \cdot \frac{b^{-7}}{b^5} \right)^3
\]

\[
\left( 2 \cdot \frac{a^{-(-2)}}{b} \cdot b^{-7-5} \right)^3
\]

\[
\left( 2 \cdot a^4 \cdot b^{-12} \right)^3
\]

Which rule should we use next?

\[
2 \cdot (a^4)^3 \cdot (b^{-12})^3
\]
\[ 8 \cdot a^{12} \cdot b^{-36} \]

Negative exponents are considered unsimplified. \[ x^{-n} = \frac{1}{x^n} \]

\[ \frac{8 \cdot a^{12}}{1 \cdot b^{36}} \]
\[ \frac{8t}{Z} - \frac{9t-7}{Z} = \frac{8t}{Z} + \frac{9t-7}{Z} \]

\[ = \frac{8t + 9t - 7}{Z} \]

\[ = \frac{17t - 7}{Z} \]

\[ = \frac{10t - 7}{Z} \]}
Converting Between Scientific and Standard Notation

Please refer to lecture notes.

AACMLSP 59 (in §5.1)
Write the following number in scientific notation:

\[ 1,176,000 \]

"break it" \[ 1.176000 = 1.171 \]

"fix it" make it bigger \[ 1.171 \times 10^6 \]

X vs. X

AACMLSP 60 (in §5.1)
Write the following number in scientific notation:

\[ 0.024 \]

"break it" \[ 0.02.4 = 2.4 \]

"fix it" make it smaller \[ 2.4 \times 10^{-2} \]
Arithmetic Operations With Scientific Notation

Please refer to lecture notes

AAAMML SP 65 (in §5.2)
The density $D$ of ordinary water is $3.12 \times 10^{-2}$ tons per cubic foot. The volume $V$ in a lake is $4.265 \times 10^{14}$ cubic feet. Use the formula

$$D = \frac{M}{V}$$

to find the mass $M$ (in tons) of the water in the lake.

$$D = \frac{M}{V} \quad D = \frac{M}{V}$$

$D \cdot V = \frac{M}{V} \cdot V$

$M = D \cdot V$

$M = 3.12 \times 10^{-2} \times 4.265 \times 10^{14}$

$M = \frac{3.12 \times 4.265 \times 10^{-2} \times 10^{14}}{10^{15}}$
$1.33068 \times 10^{13}$ (tons)
The estimated population of a country in 2007 was $2.827 \times 10^8$. The land area of the country is $3.534 \times 10^6$ square miles. Find the population density (number of people per square mile) for the country in 2007. Express your answer as a decimal rounded to two decimal places.

$$\frac{2.827 \times 10^8}{3.534 \times 10^6}$$

$$\approx 79.99 	ext{ persons per square mile}$$