Section 6.7 Financial Models

Compound Interest

(Digital Handout p. 87)

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

If Tamisha has $1000 to invest at 5% per annum compounded quarterly, how long will it be before she has $1550?

If the compounding is continuous, how long will it be?

\[ F = Pe^{rt} \]

\[ P = 1000 \]

\[ r = 5\% = 0.05 \]

\[ n = 4 \]

\[ t = ? \]

\[ F = 1550 \]

\[ t = \log_{1.0125} \left(\frac{1550}{1000}\right) \]

\[ t = \frac{\log_{1.0125}(1.55)}{\log_{1.0125}(1.0125)} \]

\[ t = 8.819767064 \text{ years} \]
56.7 Financial Models

Compound Interest

(Digital Handout p. 87)

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \quad F = Pe^{rt} \]

Ex: MM 23

If Tanisha has $1000 to invest at 5% per annum compounded quarterly, how long will it be before she has $1550?

If the compounding is continuous, how long will it be?

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \quad F = Pe^{rt} \]

\[ P = 1000 \]
\[ r = 0.05 \]
\[ n = 4 \]
\[ t = ? \]

\[ F = 1550 \]

Variable up in exponential use \( \log \)

\[ \frac{1550}{1000} = \left(1.0125\right)^{4.0t} \]

\[ \ln 1.55 = \ln 1.0125 \]

\[ \ln 1.55 - \ln 1.0125 = 4t \]

\[ 4t = \log_{1.0125} 1.55 \]

\[ 4t = 35.279 \]

\[ t = 8.819767064 \text{ years} \]
\[ P = 1000 \]
\[ r = 5\% = 0.05 \]
\[ F = 1550 \]
\[ t = ? \]

\[ F = Pe^{rt} \]
\[ 1550 = 1000 \cdot e^{0.05 \cdot t} \]
\[ \frac{1550}{1000} = e^{0.05t} \]
\[ 1.55 = e^{0.05t} \]
\[ 0.05t = \ln 1.55 \]
\[ 0.05t = 0.4382549389... \]
\[ T = \frac{0.05}{0.05} = 0.05 \]

\[ t = 8.765098614... \text{ years} \]

Continuously

\[ 8.819767064... \text{ years} \]

quarterly

Not a big difference
What rate of interest compounded annually is required to double an investment in 23 years?

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ F = 2P \] (double investment)

\[ \frac{2P}{P} \]

\[ 2P = P \left(1 + \frac{r}{1}\right)^{23} \]

\[ \frac{2P}{P} = \left(1 + r\right)^{23} \]

\[ 2 = \left(1 + r\right)^{23} \]

\[ \sqrt[23]{2} \]

\[ \sqrt[23]{2} = 1 + r \]

\[ 0.030595545 = 1 + r \]

\[ \frac{0.030595545}{1} = r \]

\[ r \approx 3.06\% \]
What will a $150,000 house cost 9 years from now if the price appreciation for homes over that period averages 8% compounded annually?

\[ P = 150000 \]
\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]
\[ t = 9 \]
\[ r = 8\% = 0.08 \]
\[ n = 1 \]
\[ F = 150000 \left(1 + \frac{0.08}{1}\right)^{9} \]
\[ F = ?? \]
\[ F = 299850.6941... \]
\[ F = \$299850.69 \]