College Algebra
§4.1 Linear Functions and Their Properties
Lecture Notes p. 46

- Increasing function
- Non-linear function
- Linear function
- Decreasing function
- Constant function
§4.1 Linear Functions and Their Properties

Lecture Notes p. 46

- Linear Function
- Increasing
- Nonlinear Function
- Decreasing
- Constant
AACMMLSP 17 and 18

Determine whether the given functions are linear or nonlinear. If linear, determine the slope.

\[
y = f(x)
\]

\[
\begin{array}{c|c}
 x & f(x) \\
 -2 & -5 \\
 0 & 1 \\
 1 & 4 \\
 4 & 13 \\
\end{array}
\]

\[
 m = \frac{y_2-y_1}{x_2-x_1}
\]

\[
 m = \frac{1-(-5)}{0-(-2)} = \frac{6}{2} = 3
\]

\[
 m = \frac{4-1}{1-0} = \frac{3}{1} = 3
\]

\[
 m = \frac{13-4}{4-1} = \frac{9}{3} = 3
\]

All match, so this is a linear function with a slope of 3.

\[
 \begin{array}{c|c}
 x & y \\
 -2 & -5 \\
 -1 & 2 \\
 1 & 1 \\
 2 & 4 \\
\end{array}
\]

\[
 m = \frac{2-(-5)}{-1-(-2)} = \frac{7}{1} = 7
\]

\[
 m = \frac{1-2}{1-(-1)} = \frac{-1}{2} = \frac{-1}{2}
\]

Not a linear function.
Suppose that the quantity supplied $S$ and the quantity demanded $D$ of T-shirts at a concert are given by the following functions where $p$ is the price.

$$S(p) = -300 + 50p$$
$$D(p) = 960 - 55p$$

(a) Find the equilibrium price and quantity for the T-shirts at this concert.

(b) Determine the prices for which quantity demanded is less than quantity supplied.

(c) What will eventually happen to the price of the T-shirts if the quantity demanded is less than the quantity supplied?

\[ (a) \quad \text{Set } S = D \]

\[ -300 + 50p = 960 - 55p \]

\[ +300 + 55p + 300 + 55p \]

\[ 105p = 1260 \]

\[ \frac{105}{105} \quad \frac{1260}{105} \]

\[ p = 12 \]

The price is $12.00
(ACMMUSP26 continued)

\[
\begin{align*}
S(p) & \quad D(p) \\
-300 + 50p & \quad 960 - 55p \\
-300 + 650 & \quad 960 - 660 \\
300 & \quad 300 \\
\end{align*}
\]

Three hundred t-shirts are the equilibrium quantity.

(b) Quantity demanded is less than quantity supplied

\[
\begin{align*}
D(p) & < S(p) \\
960 - 55p & < -300 + 55p \\
+300 + 55p & +300 + 55p \\
1260 & < 105p \\
\frac{1260}{105} & < p \\
12 & < p
\end{align*}
\]

\[p > 12\]

If the price is \$12.00, the quantity demanded will be less than the quantity supplied.

(c) The price would go down.
AEC445SP 29

Suppose that a company has just purchased a new computer for $1600. The company chooses to depreciate using the straight-line method for 4 years.

(a) Write a linear function that expresses the book value $V$ of the computer as a function of its age, $x$.

(b) What is the implied domain of the function found in part (a)?

\[ y_2 - y_1 \quad 0 - 1600 \]
\[ x_2 - x_1 \quad 4 - 0 \]
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1600}{4 - 0} = \frac{-1600}{4} = -400 \]

\[ y = mx + b \]
\[ y = -400x + 1600 \]

\[ V(x) = -400x + 1600 \]

(b) $[0,4]$ domain (implied)
The simplest cost function is the linear cost function $C(x) = mx + b$, where the y-intercept $(0, b)$ represents the fixed daily cost of operating a business and daily the slope $m$ represents the variable costs. Suppose that a small bicycle manufacturer has daily fixed costs of $2000 and each bicycle costs $100 to manufacture.

(a) Write a linear function that expresses the cost of manufacturing $x$ bicycles in a day.

$$C(x) = mx + b$$

$$C(x) = 100x + 2000$$