Section 4.1 (Start)

Vertex: \((h, k)\) Other point: \((x', y')\)

\[ y = a(x-h)^2 + k \]
\[ y' = -1(x - 4)^2 + (-9) \]

\[-13 = a(4 - 1)^2 + (-4) \]
\[ -13 = a(3)^2 - 4 \]
\[ -13 = 9a \]
\[ a = \frac{-13}{9} \]

Finding the vertex:

\[ (x, y) = (3, -9) \]

4.1 Inverse Functions

- Identify a One-to-one Function

Definition: A function is one-to-one if, for any two different inputs, you get two different outputs!

Definition: If every horizontal line intersects the graph of a function \( f \) in at most one point, then it passes the horizontal line test and \( f \) is one-to-one.
(2) Vertex: \((1, -4)\) Other point \((4, -13)\)

\[
y = a(x-h)^2 + k
\]

\[
y = -1(x-1)^2 + (-4)
\]

\[
-13 = a(4-1)^2 + (-4)
\]

\[
-13 = a(3)^2 + (-4)
\]

\[
-13 = 9a - 4
\]

\[
y = a
\]

\[
\frac{9}{9} = \frac{9}{a}
\]

\[
a = 1
\]

\[
\frac{-13}{9} = \frac{-13}{a}
\]

\[
\frac{-13}{9} = \frac{-13}{a}
\]

4.1 Inverse Functions

- Identify \(a\) One-to-one function

Definition: A function is one-to-one if, for any two different inputs, you get two different outputs.

Definition: If every horizontal line intersects the graph of a function \(f\) in at most one point, then it passes the horizontal line test and \(f\) is one-to-one.
**ALEKS Problems**

1. Yes
2. Yes
3. No
4. Yes
5. No
6. No

- **Determine Whether Two Functions are Inverse**
- **Definition of an Inverse Function**

- Let $f$ be a one-to-one function. Then $g$ is the inverse of $f$ if the following conditions are both true.

  1. $(f \circ g)(x) = x$ for all $x$ in the domain of $g$

  2. $(g \circ f)(x) = x$ for all $x$ in the domain of $f$.

- **ALEKS Problems**

  8. $f(x) = \frac{4}{x}$, $g(x) = \frac{-4}{x}$

  $$(g \circ f)(x) = \frac{-4}{f(x)} \cdot \frac{x}{4}$$

  $$(f \circ g)(x) = \frac{4}{g(x)}$$

  **Note:** $f$ and $g$ are not inverse of each other.

  $$(f \circ g)(x) = \frac{4}{x} \cdot \frac{x}{-4} = -1$$

  $$(g \circ f)(x) = \frac{-4}{\frac{4}{x}} = -x$$
2. Answer:
\[ f(x) = 4 \quad g(x) = 4x \]
\[ (f \circ g)(x) = 16x \]
\[ (g \circ f)(x) = 16x \]

They are not inverse of each other.

3. \[ f(x) = 2x + 5 \quad g(x) = \frac{x - 5}{2} \]

\[ f(g(x)) = \frac{g(x) - 5}{2} \]
\[ = \frac{\frac{x - 5}{2} - 5}{2} \]
\[ = \frac{2x + 5 - 10}{2} \]
\[ = \frac{2x - 5}{2} \]

\[ f^{-1}(x) = \frac{2x - 5}{2} \]

Find the Inverse of a Function

When we are given the equation of a one-to-one function \( f \) and asked to find its inverse \( f^{-1} \), this is how to do it:

1. Replace \( f(x) \) with \( y \).
2. Swap all \( x \)'s into \( y \)'s and all \( y \)'s into \( x \)'s.
3. Solve for \( y \).
4. Replace \( y \) with \( f^{-1}(x) \).