4.1 Linear Function and Their Properties.

Use average rate of change to identify linear functions. If the average rate of change (slope) is always the same.

Problems #3 and #8.

6) \( x \) \( \quad y = f(x) \)
   \( \begin{array}{c}
   -2 \\
   0 \\
   4 \\
   \end{array} \)
   \( \begin{array}{c}
   -5 \\
   1 \\
   1.3 \\
   \end{array} \)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ m = \frac{1 - (-5)}{0 - (-2)} = \frac{6}{2} = 3 \]

Always same slope (linear)

8) \( x \) \( \quad y \)
   \( \begin{array}{c}
   -2 \\
   -1 \\
   1 \\
   2 \\
   \end{array} \)
   \( \begin{array}{c}
   -5 \\
   2 \\
   4 \\
   \end{array} \)

\[ m = \frac{2 - (-5)}{-4 - (-2)} = \frac{7}{-2} = -\frac{7}{2} \quad \text{Not linear} \]

\[ m = \frac{4 - 1}{2 - 1} = \frac{3}{1} = 3 \]
4.1 Linear Function and Their properties

Use average rate of change to identify linear functions if the average rate of change (slope) is always the same.

Problems 17 and 48

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

17. \[ x \quad y = f(x) \]
\[ \begin{array}{c|c}
-2 & -5 \\
0 & 1 \\
1 & 4 \\
4 & 13 \\
\end{array} \]
\[ m = \frac{1 - (-5)}{0 - (-2)} = \frac{6}{2} = 3 \]
\[ m = \frac{4 - 1}{1 - 0} = \frac{3}{1} = 3 \]
\[ m = \frac{13 - 4}{4 - 1} = \frac{9}{3} = 3 \]

Always the same slope (linear)

18. \[ x \quad y \]
\[ \begin{array}{c|c}
-2 & -5 \\
-1 & 2 \\
1 & 1 \\
2 & 4 \\
\end{array} \]
\[ m = \frac{2 - (-5)}{-1 - (-2)} = \frac{7}{1} = 7 \]
\[ m = \frac{1 - 2}{1 - (-1)} = \frac{-1}{2} = -\frac{1}{2} \]
\[ m = \frac{4 - 1}{2 - 1} = \frac{3}{1} = 3 \]

Not linear
Determine whether a linear function is increasing, decreasing, or constant.

\[ \text{Increasing: slope } + \quad \text{Decreasing: slope } - \quad \text{Constant: slope } 0 \]

Just with linear functions.

Build Linear Models from Data

Problem 4.4.61

<table>
<thead>
<tr>
<th>Memory, x</th>
<th>Number of Songs</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>1450</td>
</tr>
<tr>
<td>32</td>
<td>2900</td>
</tr>
<tr>
<td>64</td>
<td>8800</td>
</tr>
<tr>
<td>128</td>
<td>11600</td>
</tr>
</tbody>
</table>

\[ a) \quad m = \frac{2900 - 1450}{32 - 46} = \frac{1450}{16} = 90.625 \]

\[ b) \quad m = \frac{8800 - 2900}{64 - 32} = \frac{5900}{32} = 90.625 \quad \text{Linear (same slope)} \]

\[ c) \quad m = \frac{11600 - 8800}{128 - 64} = \frac{2800}{64} = 90.625 \]

Slope: 90.625

Points: (0,0) (16, 90.625) (46, 1450)

\[ y - y_1 = m(x - x_1) \]

\[ y - 0 = 90.625(x - 0) \]

\[ y = 90.625 \]

\[ n = 90.625 \]
Slope = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{AN}{AX} = \frac{90.625}{1}

Domain = [1, \infty)

1 GB of Memory holds 90.625 songs

\begin{align*}
\text{number of songs} & \quad 12000 \\
11000 & \\
10000 & \\
9000 & \\
8000 & \\
7000 & \\
6000 & \\
5000 & \\
4000 & \\
3000 & \\
2000 & \\
1000 & \\
\end{align*}
Build linear models from verbal descriptions.

For example, economic supply and demand curves can be approximated by lines.

Suppose that the quantity supplied $S$ and quantity demanded $D$ of T-shirts at a concert are given by the following functions where $p$ is the price:

$$S(p) = -300 + 50p$$
$$D(p) = 960 - 55p$$

1. **Equilibrium Supply = Demand**
   
   $$S(p) = D(p)$$
   
   $$-300 + 50p = 960 - 55p$$
   
   $$105p = 1260$$
   
   $$p = 12$$

2. **$D(p) \leq S(p)$**

   $$960 - 55p \leq -300 + 50p$$
   
   $$960 + 300 \leq 55p + 50p$$
   
   $$1200 \leq 105p$$
   
   $$12 \leq p$$
   
   $$p \geq 12$$

3. Lower the price until it fits to equilibrium, $\$12$.
Company purchased a new computer for $1600. The company chooses to depreciate using the straight-line method for $y$ years.

The value of the computer decreases linearly with age. The initial value is $1600$, and the slope $m$ can be calculated using the points $(4,0)$ and $(0,1600)$.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ m = \frac{1600 - 0}{4 - 4} \]

\[ m = \frac{1600}{4} \]

\[ m = -400 \]

The equation of the line is:

\[ y - y_1 = m(x - x_1) \]

\[ y - 0 = -400(x - 4) \]

\[ y = -400x + 1600 \]

\[ \sqrt{(x)} = -400x - 1600 \]

**a**

\[ y = y_1 - m(x - x_1) \]

\[ y = 0 = -400(x - 4) \]

\[ y = -400x + 1600 \]

\[ \sqrt{(x)} = -400x - 1600 \]

**b** Domain $= [0,4]$  

**c** $C(x) = mx + b$

$C(x) = 400x + 2000$

$m = 400$

$b = 2000$

$m$ represents the variable cost  

$b$ is the fixed cost