Financial Models

7 days until Chapter 6 test
(6.4, 6.5, 6.6, 6.7, 6.8)

Compound Interest Formula

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

- \( F \): final amount
- \( P \): principal amount
- \( r \): annual interest rate
- \( n \): number of times interest is compounded per year
- \( t \): time in years

Interest rates are expressed as a percentage per year but must be rewritten as a fraction.

VS.

Continuously Compounded Interest Formula

\[ F = Pe^{rt} \]

#39 a) \$1000 to invest, five-year period, 5% interest rate, compounded quarterly

\[ F = 1000 \left(1 + \frac{0.05}{4}\right)^{4 \cdot 5} = 1000 \left(1 + 0.0125\right)^{20} \]

\[ 1550 = 1000 \left(1 + \frac{0.05}{4}\right)^{20} \]

Solve for \( t \)

when solving for something in the exponent use a log
7 days until Chapter 6 test

(6.4, 6.5, 6.6, 6.7, + 6.8)

Compound Interest Formula

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

* interest rate it is expressed as a % but must be re-written as a fraction

\[ \text{Cumulative Sums} \]

VS.

Continuously Compound Interest Formula

\[ F = Pe^{rt} \]

#39 a) $1000 to invest, future amount $1550

5% interest rate, compounded quarterly, only tells time

\[ F = 1550 \]
\[ P = 1000 \]
\[ r = 5\% = 5(0.01) = 0.05 \]
\[ n = 4 \]
\[ t = ? \]

Solve for \( t \) when solving for something in the exponent

use a log
\[ 1550 = 1000 \left(1 + \frac{0.05}{4}\right)^{4t} \]

Use convert:

\[ \log 1550 = \log \left(1 + \frac{0.05}{4}\right)^{4t} \]

\[ \log 1550 - \log 1000 = \log_{1.0125}^{4t} \]

\[ \frac{1550}{1000} = \left(1.0125\right)^{4t} \]

\[ 1.55 = \left(1.0125\right)^{4t} \]

\[ \log_{1.0125} 1.55 = \frac{4t}{4} \]

\[ \log_{1.0125} 1.55 = t \]

Change of base rules:

\[ t \approx 8.82 \text{ years} \]

\[ \frac{\log_{1.0125} 1.55}{\log_{1.0125} \left(\frac{1}{4}\right)} = 8.82 \]
b. compounded continuously

\[ F = P e^{rt} \]

\[ F = 1000 e^{0.05t} \]

\[ \frac{1550}{1000} = \frac{1000e^{0.05t}}{1000} \]

\[ \sqrt{1.55} = e^{0.05t} \]

\[ \ln 1.55 = 0.05t \]

\[ \frac{\ln 1.55}{0.05} = t \]

\[ t \approx 8.77 \text{ years} \]
6.7.21

Find the principal needed now to get $650 in 4 years at 3% interest.

\[ P = 50 \]
\[ P = ? \]
\[ n = 1 \times 4 = 4 \]
\[ \frac{3}{4} = \frac{4}{4} = 1.75 \]
\[ t = 3 \% = 0.03 \]
\[ F = Pe^{nt} \]
\[ 50 = Pe^{1.75 \times 0.03} \]
\[ 50 = Pe^{0.0525} \]
\[ \frac{50}{e^{0.0525}} = P \]
\[ \frac{50}{e^{0.0525}} = P \]

\[ P = \$447.44 \]

6.7.19

On your own

Find the
Determine the Rate of Interest or the Time Required to Double a Lump Sum of Money.

6.7.31

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

Given:
\[ F = 2P \]
\[ P = P \]
\[ n = 4 \]
\[ t = \frac{23}{23} \]

\[ 2P = P \left(1 + \frac{r}{1}\right)^{23} \]
\[ 2 = \left(1 + \frac{r}{1}\right)^{23} \]
\[ 2^{\frac{1}{23}} = 1 + r \]
\[ 2^{\frac{1}{23}} - 1 = \frac{1}{23} \cdot r \]

\[ r \approx 0.0305955 \]

\[ r \approx 3.06\% \]