2.4 Circles

Distance formula:
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Definition:
\[ (x-h)^2 + (y-k)^2 = r^2 \quad \Rightarrow \text{standard form} \]

Theorem:
\[ x^2 + y^2 = r^2 \]

Definition:
\[ x^2 + y^2 = 1 \quad \Rightarrow \text{the unit circle} \]

2.4, 7

\[
\begin{align*}
&f(x, y) = 6 \quad (3, -5) \quad \text{center/standard form} \\
&h = 3 \quad k = -5 \\
&(x - 3)^2 + (y + 5)^2 = r^2 \\
&(x - 3)^2 + (y + 5)^2 = 36
\end{align*}
\]

2.4, 25

Identify the first step in finding the radius and center:
\[
\begin{align*}
&2(x - 3)^2 + 2y^2 = 8 \\
&(x - h)^2 + (y - k)^2 = r^2
\end{align*}
\]
2.4 circles

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distance formula
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

definition
\[ (x-h)^2 + (y-k)^2 = r^2 \iff \text{standard form} \]

theorem
\[ x^2 + y^2 = r^2 \]

definition
\[ x^2 + y^2 = 1 \implies \text{the unit circle} \]

2.4.7
\[ f = 6 \]
\[ \left( \frac{3}{6}, \frac{-5}{6} \right) = \text{center}\text{, standard form} \]
\[ h = \frac{3}{2} \quad k = \frac{-5}{2} \]
\[ (x-h)^2 + (y-k)^2 = r^2 \]
\[ (x-3)^2 + (y-(-5))^2 = 6^2 \]
\[ (x-3)^2 + (y+5)^2 = 36 \]

2.4.26 
Identify the first step in finding the radius and center.
\[ 2(x-5)^2 + 2y^2 = 8 \]
\[ (x-h)^2 + (y-k)^2 = r^2 \]
\[ \frac{2(x-5)^2}{2} + \frac{2y^2}{2} = \frac{8}{2} \]
\[ \frac{x(x-5)^2}{8} + \frac{2y^2}{2} = \frac{8}{2} \]
\[ (x-5)^2 + y^2 = 4 \]

\[ h=5 \quad k=0 \quad r=2 \quad (5,0) \]

2.4.17

Graph circle \( r = 5 \) center \((-3, -4)\)

\[ \frac{(x - h)^2}{25} + \frac{(y - k)^2}{25} = 1 \]

\[ \frac{(x - (-3))^2}{25} + \frac{(y - (-4))^2}{25} = 1 \]

\[ (x + 3)^2 + (y + 4)^2 = 25 \]
General Form of a Circle

\[(x-3)^2 + (y+5)^2 = 36\]
\[(x-3)(x-3) + (y+5)(y+5) = 36\]

\[x^2 - 6x + 9 + 10y + 25 = 36\]
\[x^2 + y^2 - 6x + 10y - 2 = 0 \rightarrow \text{general form}\]

\[x^2 + y^2 + ax + by + c = 0 \rightarrow \text{general form}\]

- Can get circle or dot or no graph
2.4.17

General form equation of a circle with radius 5 and center (-3, -4):

\[ x^2 + y^2 + ax + by + c = 0 \]

\[(x+3)^2 + (y+4)^2 = 25 \]

\[(x+3)(x+3) + (y+4)(y+4)\]

\[x^2 + 6x + 9 + y^2 + 8y + 16 \]

\[x^2 + 6x + 9 + y^2 + 8y + 16 = 25\]

\[x^2 + y^2 + 6x + 8y + 9 + 16 - 25 = 0\]

\[x^2 + y^2 + 6x + 8y + 0 = 0\]

\[x^2 + y^2 + ax + by + c\]

\[a = 6 \quad b = 8 \quad c = 0\]
for the equation \(2x^2 + 2y^2 - 16x + 8y - 10 = 0\) find

a) the center \((h, k)\) and radius \(r\)

\[
\frac{2x^2 + 2y^2 - 16x + 8y - 10}{2} = 0
\]

\[
x^2 + y^2 - 8x + 4y - 5 = 0
\]

\[
(4, -2) \quad r = 5
\]

b) graph

completing the square

\[
\frac{1}{2}x^2 + \frac{1}{2}y^2 - 8x + 4y - 5 = 0
\]

group all terms w/ the same variables

- put in decreasing order
- move any #s to right side

\[
x^2 - 8x + y^2 + 4y = 5
\]

\[
x^2 - 8x + 16 + y^2 + 4y + 4 = 5 + 16 + 4
\]

- put #s in blanks
- \(1/2 \times 8 + \) square it
- or the 2nd value divide by 2 \times square

\[
x^2 - 8x + 16 + y^2 + 4y + 4 = 25
\]

\[
(x - 4)^2 + (y + 2)^2 = 25
\]

\[
(x - 4)(x - 4) + (y + 2)(y + 2) = 25
\]

C) find any intercepts

touches \(x\) or \(y\) intercepts

any \(x\) intercept has a \(y\) coordinate of zero

any \(y\) intercept has an \(x\) coordinate of zero

1) to find \(x\) intercepts let \(y = 0\)

solve for \(x\)

\[
2x^2 + 2(0)^2 - 16x + 8(0) - 10 = 0
\]

2) to find \(y\) intercepts let \(x = 0\)

solve for \(y\)

\[
2(0)^2 + 2y^2 - 16(0) + 8y - 10 = 0
\]