No Test Today

§ 4.4 Building Quadratic Models

From Verbal Descriptions

(pp. 64-70)

\[ p := \text{price} \]
\[ x := \text{quantity sold} \]

Demand equation:
\[ x = -8p + 116 \quad 0 \leq p \leq 30 \]

(a) Express \( R \) as a function of \( x \).

R = x - p

\[ R = x - \left( -\frac{1}{8}x + 22 \right) \]

\[ R(x) = x \left( -\frac{1}{8}x + 22 \right) \]

\[ R(x) = -\frac{1}{8}x^2 + 22x \]

\[ \frac{8p}{8} = 176 - x \]

\[ p = \frac{176 - x}{8} \]

\[ p = 22 - \frac{x}{8} \]

\[ p = \frac{1}{8}x + 22 \]
§4.4 Building Quadratic Models
From Verbal Descriptions (pp. 64-67)

4. \( p = \text{price} \)
\( x = \text{quantity sold} \)

Demand equation: \( x = -8p + 176 \quad 0 \leq p \leq 30 \)

(a) Express \( R \) as a function of \( x \)

\[
R = x \cdot p
\]

must get rid of \( p \)

\[
8p = 176 - x
\]

\[
\frac{8p}{8} = 176 - \frac{x}{8}
\]

\[
p = \frac{176}{8} - \frac{x}{8}
\]

\[
p = 22 - \frac{x}{8}
\]

\[
p = -\frac{1}{8}x + 22
\]

\[
R(x) = x \left( -\frac{1}{8}x + 22 \right)
\]

\[
R(x) = -\frac{1}{8}x^2 + 22x
\]
(b) "Implied domain"

Smallest $x$ (quantity sold): $x = 0$

Largest $x$?

Set price to "free"

$\text{demand } x = -0.8\,p + 176$

$x = -0.8(0) + 176$

$x = 176$

$[0, 176]$ (domain)

(c) What if revenue if 144 units are sold?

What is $R$ if $x = 144$?

$R(x) = -\frac{1}{8}x^2 + 22x$

$R(144) = -\frac{1}{8}(144)^2 + 22(144)$

$= 576$

(a) What $x$ maximizes revenue?

$R(x) = -\frac{1}{8}x^2 + 22x$

$y = -\frac{1}{8}x^2 + 22x$

$\text{WINDOW}$

$X_{min} = 0$ $\	ext{use ZoomFit to automatically determine } Y_{min}$ and $Y_{max}$.

$X_{max} = 176$
\[
(87.999993, 9.68)
\]

999999 rounded up (exact)
000000 rounded down (exact)

\[
(88, 9.68) \quad \times \quad R(x)
\]

Quantity Sold \quad Revenue

(d) What \( x \) maximizes revenue? 89 units
What is the maximum revenue? \$968

(e) What price maximizes revenue?

\[
R = x \cdot p
\]

\[
9.68 = 88 \cdot p
\]

\[
\frac{9.68}{88} = \frac{p}{88}
\]

11 \( = p \)

\( p = \$11.00 \)
What price should the company charge to earn at least $840 in revenue?

Intersect

1st curve \( \cap \) enter
2nd curve \( \rightarrow \) enter
axis enter

\[(56, 840) \quad (120, 840)\]

\[R = x \cdot p\]

\[
\begin{align*}
840 &= 56p \\
R &= \frac{840}{56} \\
15 &= p
\end{align*}
\]

If \(7 \leq p \leq 15\), then

Revenue \(\geq 840\)
(a) Express the area $A$ of the fence as a function of the width $W$.

$$P = 2W + 2L$$
$$280 = 2W + 2L$$

Replace the "L" out of the equation

$$\frac{280}{2} - \frac{2W}{2} = \frac{2L}{2}$$

$$140 - W = L$$

(b) For what $W$ is the area $A$ largest?

(c) What is the maximum area $A$?
Physics (Type 3)

(a) At what horizontal dist. \( x \) is the projectile at its maximum?

Since \( h(x) \) is a parabola, the max. is at the vertex.

We want \( x \)-coord of vertex.

\[
\left( \frac{-b}{2a}, \frac{c - b^2}{4a} \right)
\]

part (a)

\[
a = \frac{-32}{55^2}, \quad b = 1
\]

\[
\frac{-b}{2a} \quad \frac{3025}{64}
\]

(b) Max. Height

\[
(\text{y-Coord of vertex})
\]

\[
c - \frac{b^2}{4a} \rightarrow \frac{27345}{128}
\]
(c) When projectile hits water, \( h = 0 \)

\[ h(x) = \frac{-32x^2}{55^2} + 12x + 190 \]

\[ 0 = \frac{-32x^2}{55^2} + 12x + 190 \]

\[ a = \frac{-32}{55^2}, \quad b = 12, \quad c = 190 \]

\[ 0 = ax^2 + bx + c \quad \text{quad. eqn} \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Enter quadratic form in calc.

Using \( + \), we get a negative answer.

Using \( - \) we get real answer, 180 feet.

(d) Graph the equation. \( h(x) = \frac{-32x^2}{55^2} + 12x + 190 \)

(e) 151 feet

\[ y = ax^2 + bx + c \]

\( 151 \text{ feet} \)

\( (150.90853, 100) \)