July 25th, 2016

Section 6.8: Exponential Growth and Decay Models
(Lect Notes pp 121-122)

**Find Equations**

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Amount of medicine in bloodstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 hours</td>
<td>743 mg</td>
</tr>
<tr>
<td>1 h</td>
<td>81 mg</td>
</tr>
<tr>
<td>2 h</td>
<td>9 mg</td>
</tr>
<tr>
<td>3 h</td>
<td>1 mg</td>
</tr>
<tr>
<td>4 h</td>
<td>0.33 mg</td>
</tr>
<tr>
<td>5 h</td>
<td>0.1 mg</td>
</tr>
<tr>
<td>6 h</td>
<td>0.03 mg</td>
</tr>
<tr>
<td>7 h</td>
<td>0.01 mg</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Time</th>
<th># of Tadpoles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 days</td>
<td>8 Tadpoles</td>
</tr>
<tr>
<td>3 days</td>
<td>17 Tadpoles</td>
</tr>
<tr>
<td>6 days</td>
<td>40.5 Tadpoles</td>
</tr>
<tr>
<td>9 days</td>
<td>68.75 Tadpoles</td>
</tr>
</tbody>
</table>

b) To get the multiplier \( \frac{12}{8} = 1.5 \) or \( \frac{18}{12} = 1.5 \)

If the # is not in the table you can not use the table

\[ \boxed{\text{NO}} \]


July 25th, 2016

Section 6.8: Exponential Growth and Decay Models
(Lect Notes pp. 121-124)

Find Equations

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Amount of Medicine in bloodstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>0h</td>
<td>243 mg</td>
</tr>
<tr>
<td>1h</td>
<td>81 mg</td>
</tr>
<tr>
<td>2h</td>
<td>27 mg</td>
</tr>
<tr>
<td>3h</td>
<td>9 mg</td>
</tr>
<tr>
<td>4h</td>
<td>3 mg</td>
</tr>
<tr>
<td>5h</td>
<td>1 mg</td>
</tr>
<tr>
<td>6h</td>
<td>(\frac{1}{3}) mg</td>
</tr>
<tr>
<td>7h</td>
<td>(\frac{1}{9}) mg</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Time (days)</th>
<th># of Tadpoles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 days</td>
<td>8 Tadpoles</td>
</tr>
<tr>
<td>+ 3 days</td>
<td>17 Tadpoles</td>
</tr>
<tr>
<td>+ 3 days</td>
<td>18 Tadpoles</td>
</tr>
<tr>
<td>+ 3 days</td>
<td>27 Tadpoles</td>
</tr>
<tr>
<td>+ 3 days</td>
<td>40.5 Tadpoles</td>
</tr>
</tbody>
</table>

\[ \frac{12}{8} = 1.5 \quad \frac{18}{12} = 1.5 \]

To get the multiplier, \( \frac{12}{8} \) = 1.5, \( \frac{18}{12} \) = 1.5

If the # is not in the table you cannot use the table. **No**
Long ways

Exponential law

\[ A(t) = A_0 e^{kt} \]

\( A_0 \) &= "a naught" or "a sub zero" (Amount at \( t = 0 \))
\( k > 0 \) &= growth
\( k < 0 \) &= decay
\( m = \text{multiplier} \), \( d = \text{divider} \)

\[ k = \frac{\ln(m)}{a} \quad \text{or} \quad k = \frac{\ln\left(\frac{1}{d}\right)}{a} \]

6.85 A population of a colony of mosquitoes obey the law of uninhibited growth.

The table below relates time and the number of mosquitoes.

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Amount of mosquitoes</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000 mosquitoes</td>
<td>( \frac{1900}{1000} = 1.9 )</td>
</tr>
<tr>
<td>2</td>
<td>1900 mosquito</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3610 mosquitoes</td>
<td>( \frac{3610}{1900} = 1.9 )</td>
</tr>
<tr>
<td>6</td>
<td>6859 mosquitoes</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>13032.1 mosquitoes</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>24760.99 mosquitoes</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>47045.881 mosquitoes</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>89387.1739 mosquitoes</td>
<td></td>
</tr>
</tbody>
</table>

What is the size of the colony after 6 days?

6859 mosquitoes
How long is it until there are 1,000,000 mosquitoes?

We can use the formula:

\[ A(t) = A_0 e^{kt} \]

where:

\[ A_0 = 1000 \]

\[ k = \frac{\ln 1.9}{a} = \frac{\ln 1.9}{2} \]

Amount:

\[ A(t) = 1000 e^{\left(\frac{\ln 1.9}{2}\right)t} \]

\[ \frac{700000}{1000} = 1000 e^{\frac{\ln 1.9}{2} t} \]

\[ \Rightarrow t_0 = e^{\frac{\ln 1.9}{2} t} \]

\[ \log_e t_0 = \frac{\ln 1.9}{2} \]

\[ \log_e e^{\frac{\ln 1.9}{2}} = \frac{\ln 1.9}{2} \]

\[ \frac{2 \ln t_0}{\ln 1.9} = t \]

\[ t = \frac{2 \ln t_0}{\ln 1.9} \approx 13.238 \text{ days} \]

\[ t \approx 13 \text{ days 6 hours} \]
6.8.3 Strontium 90 is a radioactive material that decay according to the function \[ A(t) = A_0 e^{-0.0244t} \], where \( A_0 \) is the amount present and \( A \) is the amount present at time \( t \) (in years). Assume that a scientist has a sample of 800 grams of Strontium 90.

\[ A(t) = A_0 e^{-0.0244t} \]

<table>
<thead>
<tr>
<th>Time</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>800 gams</td>
</tr>
</tbody>
</table>

a) What is the decay rate of the Strontium 90

\[ -0.0244 = -2.44\% \]

b) How much strontium 90 is left after 20 years?

If \( t = 20 \) what is \( A(t) \)?

\[ A(t) = 800 e^{-0.0244t} \]

\[ A(20) = 800 e^{-0.0244(20)} \]

\[ A(20) \approx 494.08 \text{ grams} \]
2) What is the half-life of Strontium 90?

Definition: Half-life is the length of time needed for a material to decay such that only half of the material is left.

Half of 800 is 400
Half of Ao is \( \frac{Ao}{2} \)

\[ A(t) = Ao e^{-0.0244t} \]

\[ \frac{1}{2} Ao = Ao e^{-0.0244t} \]

\[ \frac{1}{2} = e^{-0.0244t} \]

\[ \log_{e} \left( \frac{1}{2} \right) = -0.0244t \]

\[ \log_{e} \left( e^{0.0244t} \right) = -0.0244t \]

\[ \ln \left( \frac{1}{2} \right) = -0.0244t \]

\[ \frac{-\ln \left( \frac{1}{2} \right)}{-0.0244} = t \approx 28.41 \text{ years} \]
The half-life of carbon-14 is 5600 years. If a piece of charcoal made from the wood of a tree shows only 64\% of the carbon-14 expected in living matter, when did the tree die?  

For a half-life problem the amount is \( \frac{1}{2} A_0 \).

Half-life is 5600 years.

If you know \( k \) → find half-life

If you know half-life → find \( k \)

\[ A(t) = A_0 e^{kt} \]

\[ \frac{1}{2} A_0 = A_0 e^{kt} \]

\[ 0 \rightarrow \frac{1}{2} A_0 = A_0 e^{k(5600)} \]

\[ \frac{1}{2} = e^{k(5600)} \]

\[ \frac{\ln \frac{1}{2}}{5600} = k \]

\[ \frac{\ln \frac{1}{2}}{5600} = k \]

\[ \ln \frac{1}{2} = 5600k \]

\[ \frac{\ln \frac{1}{2}}{5600} \cdot \frac{5600}{5600} = k \]

\[ k = \frac{\ln \frac{1}{2}}{5600} \]

\[ A(t) = A_0 e^{kt} \]

\[ A(t) = A_0 e^{\frac{\ln \frac{1}{2}}{5600} \cdot t} \]
\[
\frac{0.64 \ A_0}{A_0} = 0.64e^{\frac{\ln 0.64}{5600} \ t} = 0.64
\]

\[0.64 = e^{\frac{\ln 0.64}{5600} \ t}\]

\[
\log_e 0.64 = \frac{\ln 0.64}{5600} \ t
\]

\[
0.64 = 5600 \frac{\ln 0.64}{\ln 0.5}
\]

\[t = \frac{5600 \ln 0.64}{\ln 0.5} \approx 3606 \text{ years}
\]

6.8

The size \( P \) of a certain insect population at time \( t \) (in days) obeys the function \( P(t) = 300e^{0.04t} \).

\[A(t) = A_0e^{kt}\]

a) Determine the number of insects at \( t = 0 \) day:

\[300 \text{ insects}\]

b) What is the growth rate of the insect population?

\[k = 0.09 \quad \text{9% per day}\]
c) What is the population after 10 days?

\[ P(t) = 300e^{0.09t} \]

\[ P(10) = 300e^{0.09 \cdot 10} \]

\[ P(10) = 737 \text{ insects} \]

d) When will the insect population reach 420?

\[ P(t) = 420 \]

\[ \frac{420}{300} = \frac{300e^{0.09t}}{300} \]

\[ 1.4 = e^{0.09t} \]

\[ \ln 1.4 = 0.09t \]

\[ \frac{\ln 1.4}{0.09} = t \approx 3.74 \text{ days} \]

e) When will the population double?

\[ 2A_0 = P(t) = 2 \cdot 300 = 600 \]

\[ P(t) = 300e^{0.09t} \]

\[ 600 = \frac{300e^{0.09t}}{300} \]

\[ 2 = e^{0.09t} \]
\[ a = e^{0.09t} \]

\[ \log_e a = 0.09t \]

\[ \frac{\log_e 2}{0.09} = t = 7.70 \text{ days} \]

6.94 The population of a southern city follows the exponential law. If the population doubled 10 years over 15 months and the current population is 70,000, what will the population be 3 years from now?

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>70000</td>
</tr>
<tr>
<td>1.25</td>
<td>140000</td>
</tr>
</tbody>
</table>

\[ k = \frac{\ln 2}{0.25} \]

Amount

\[ A(t) = A_0e^{kt} \]

\[ A(4) = 70000e^{\frac{\ln 2}{0.25} \cdot 4} \]

\[ A(3) = 70000e^{\frac{\ln 2}{0.25} \cdot 3} \]

\[ A(3) \approx 369,462.215 \]

\[ \approx 369,462 \]