§6.6 Logarithmic and Exponential Equations (lect. Notes p. 90)

General tips for success:
- Form study groups
- Complete study plan problems in addition to your homework.

Specific tips for success:
1. The equation you will use the most are
   \[ x = a^y \iff y = \log_a x \]
   \[ \log_a M = \log_a N \iff M = N \]

2. Identify the domain, the last step is to throw away any "extraneous" solutions that are outside of the domain.

3. \[ \log_a M + \log_a N = \log_a (MN) \] and \[ \log_a M - \log_a N = \log_a \left( \frac{M}{N} \right) \]

Problem #10
\[ \log_2 \left( \frac{x-4}{16} \right) + \log_8 16 = 2 \]
\[ x - 4 > 0 \]
\[ x > 4 \]
\[ x > 4 \quad \text{domain} \]
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Problem #10
\[ 2 \sqrt[3]{x - 4} + \log_8 16 = 2 \]

\[ x - 4 > 0 \]
\[ +4 +4 \]
\[ x > 4 \text{ domain} \]
\[
\log_8 (x-4)^2 + \log_8 16 = 2
\]

\[
\log_8 [(x-4)^2 16] = 2
\]

\[
\text{Let } y = \log_8 x \iff x = 8^y
\]

\[
\begin{array}{l}
\square = \bigcirc \iff \log_{\bigcirc} \square = \bigtriangleup \\
8^2 = (x-4)^2 16
\end{array}
\]

Intermediate algebra:

\[
X = 6 \text{ or } \sqrt{x-4} \leftarrow \text{No in the domain extraneous}
\]

Check your work:

\[
2\log_8 (6-4) + \log_8 16 = 2
\]

\[
2^2 = 2 \text{ (calc)}
\]
\[ \log_a \left( \frac{x-8}{x+5} \right) = \log_a \left( \frac{x-4}{x+6} \right) \]

Find the domain:
\[ x - 8 > 0 \quad \text{and} \quad x + 5 > 0 \quad \text{and} \quad x - 4 > 0 \quad \text{and} \quad x + 6 > 0 \]
\[ x > 8 \quad \text{and} \quad x > -5 \quad \text{and} \quad x > 4 \quad \text{and} \quad x > 6 \]
Thus, \( x > 8 \) is the domain.

\[ \frac{\log \left( \frac{x-8}{x+5} \right)}{\log a} = \frac{\log \left( \frac{x-4}{x+6} \right)}{\log a} \]

Multiply both sides by \( \log a \):
\[ \log \left( \frac{x-8}{x+5} \right) = \log \left( \frac{x-4}{x+6} \right) \]

Intermediate algebra:
\[ \frac{x - 8}{x + 6} = \frac{x - 4}{x + 6} \]

Extraneous:
\[ x = \frac{-28}{3} \]

\[ \rightarrow \text{No Solution!!} \]
23) Specific 5, 8

\[ 6^{1-6x} = 7^x \]

51) Take the log of both sides to solve the problem.

Recall:

\[ 5^{x^2 - 60} = 25 \]
\[ 5^{x^2 - 60} = (5^2)^x \]
\[ 5^{x^2 - 60} = 5^{2x} \]

Choose one base:
- base 6
- base 10
- base e

\[ \log_6 6^{1-6x} = \log_6 7^x \]
\[ 1 - 6x = \log_6 7^x \]
\[ \log_6 6^{1-6x} = \log_6 7^x \]
\[ \log_7 (1-6x) = \log_7 x \]
\[ \log_{10} (1-6x) = \log_{10} x \]

\[ (1-6x) \log_6 6 = x \log_7 7 \]
\[ \log_6 (1-6x) = \log_7 x \]

\[ 6^{1-6x} = 7^x \]

Rule 9:

\[ (1-6x) \log_6 6 = x [\log_7 x] \]

Distribute:

\[ \log_6 (1-6x) = x \log_7 x \]
\[ \text{Move: } \quad \ln 6 = x \ln 7 + 6 \ln 6 \]
\[ \text{Factor: } \quad \ln 6 = x (\ln 7 + 6 \ln 6) \]
\[ \text{Divide: } \quad \frac{\ln 6}{\ln 7 + 6 \ln 6} = x \]

Steps for this Problem:
- Rule 9
- Distribute
- Move
- Factor
- Divide

"9_DMF"