Finding Vertical Asymptotes from a Rational Function

Factor both numerator and denominator and reduce to lowest terms.

\[ \frac{x^3 - 8}{x^2 - x - 2} \]

\[ R(x) = \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+1)} \]

\[ a = x - 2, \quad b = 2 \]

There will be a vertical asymptote at each value of \( x \) where the reduced denominator polynomial equals zero.

Finding Domain vs. Finding Vertical Asymptotes

\[ \text{Quiz today!!} \]
Finding Vertical Asymptotes
From a Rational Function Eqn

- Factor both numerator and denominator and reduce to lowest terms.

\[ \frac{x^3 - 8}{x^2 - x - 2} \]

\[ R(x) = \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+1)} \]

\[ R(x) = \frac{x^2 + 2x + 4}{x+1} \]

\[ \frac{x^3 - 8}{GCF} = 1 \]

\[ \frac{x^3 - a^3}{a^3 - b^3} = \frac{(a-b)(a^2 + ab + b^2)}{a = x, b = 2} \]

\[ (x-2)(x^2 + 2x + 4) \]

Factored form

\[ x^2 - x - 2 \]

\[ (x-2)(x+1) \]

There will be a vertical asymptote at each value of \( x \) where the reduced denominator polynomial equals zero.

[5.2.1] Finding Domain vs. Finding Vertical Asymptotes
Same

Both involve denom = \( \neq 0 \)

Different

For domain, it's the original denominator.
For vert. asy, its reduced denominator.

For domain, solutions are "bad" numbers which are removed from domain.
For vert. asym. solutions are equations of the vert. asym.

For domain we IGNORE the numerator.
For vert. asy. we consider the numerator.

Finding Horizontal and/or Oblique Asymptotes From a Rational Function

You will only find ONE of these, hence will NEVER be both.
- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is \( y = 0 \).

\[
R(x) = \frac{3x}{x + 13} \quad T(x) = \frac{x^3}{x^4 - 16}
\]

\[
G(x) = \frac{x^4 - 256}{2x^2 - 8x} \quad Q(x) = \frac{x^3 - 27}{x^2 + x - 12}
\]

\[
\begin{align*}
R(x) &= \frac{3x}{x + 13} \quad \text{degree 1} \\
&\quad \text{Since } 1 > 1, \text{ case } B \\
&\quad \text{ (lecture notes pg 65)} \\
&\quad b = \frac{3}{1} \Rightarrow [y = 3] \\
y &= b
\end{align*}
\]

\[
T(x) = \frac{x^3}{x^4 - 16} \quad \text{Since } 3 < 4, \text{ case } A \\
y &= 0
\]

\[
G(x) = \frac{x^4 - 256}{2x^2 - 8x} \quad \text{Since } 4 \text{ is 2 more than 2, case C, none}
\]
\[ Q(x) = \frac{x^3 - 27}{x^2 + x - 12} \]

Since 3 is one more than 2, polynomial long division

1. \[ \frac{1^{st} \text{ term dividend}}{1^{st} \text{ term divisor}} \] 
   \[ \frac{x^3}{x^2} \]

2. Take result and mult by entire divisor
   \[ (x^2 + x - 12)x^3 + 10x^2 + 0x - 27 \]
   \[ -x^3 - x^2 + 12x \]
   \[ 0 - x^2 + 12x - 27 \]
   \[ + x^2 + x + 12 \]
   \[ 0 13x - 39 \]

3. Place result beneath dividend, then...
   "draw the line, change the sign, then combine"

4. Bring down the next term of the original dividend.
   \[ \frac{x^3}{x^2} = -x \]

5. Repeat.
   \[ x(x^2 + x - 12) \]
   \[ -x^2 - x + 12 \]

- As long as there is a remainder, our answer is \( x - 1 \).