Test in 7 days!

5.4 cont. pg. 5b

5. David has available 280 yards of fencing and wishes to enclose a rectangular area. 

(a) Smallest and largest values for \( W \)?

Smallest width = 0
Largest width = 140 (2 perimeter)

(b) Express the area \( A \) of the rectangle as a function of the width \( W \) of the rectangle.

Goal: \( A(W) \)

Start \( A = LW \) needs to go away therefore solve this for \( L \)

Only variable is \( W \)

\( p = 2L + 2W \)

\( 280 = 2L + 2W \)
§ 4.4 cont. pg. 56

David has available 280 yds of fencing and wishes to enclose a rectangular area.

\[ \text{Length} \quad \text{width} \quad W \quad L \]

(a) Smallest and largest values for \( W \)?

Smallest width: 0

Largest width: 140 (since perimeter)

(b) Express the area \( A \) of the rectangle as a function of the width \( W \) of the rectangle.

Goal: \( A(W) \)

Only variable is \( W \)

\[ p = 2L + 2W \]

Start \( A = LW \)

needs to go away therefore solve this for \( L \)
(b) cont. 

\[
\begin{align*}
280 &= 2w + 2w^2 \\
&= 2w + 2w^2 \\
&= 2w(nw) \\
\Rightarrow \quad \frac{280 - 2nW}{a} &= \frac{L}{a} \\
\left(140 - W = L, \text{ substitute into}\right) \quad A &= L \cdot W
\end{align*}
\]

\[
A = L \cdot W \quad \Rightarrow \quad A = (140 - W) \cdot W
\]

\[
A(W) = 140W - W^2 \quad \text{or} \quad A(W) = -W^2 + 140W
\]

(c) For what value of \( W \) is the area largest?

- Use calculator. Set \( X_{\text{min}} = 0 \), \( X_{\text{max}} = 140 \), then GProb.

- \( X \) is width, \( y \) is area. \( X = 70 \), \( y = 4900 \)

Area is measured in “squares” \( \rightarrow \text{sqft, ft}^2 \)

70 yards
(d) What is the maximum area?
\[ A = 4800 \text{ sq yards or yd}^2 \]

"Must watch pencast on this!!"

track and field problem
and physics problem
are pencast!

§ 5.1 Polynomials, Functions, and Models

- If you can put a function in the form:
  \[ f(x) = \square + \square x + \square x^2 + \square x^3 + \ldots \]

3, 5, 7, 1, -1, 2, 0 \rightarrow \text{rational (integers)}

3, 5, 7, 1, -1, 2, 0 \rightarrow \text{rational (integers)}

real numbers only, \[ i^2 = \text{imaginary} \]

16 and 17

\[ g(x) = \frac{11x^2}{8} \]

\[ f(x) = 4 - \frac{5}{x} \]
\( g(x) = \prod_{k=0}^{\infty} x^2 \) a polynomial?

\[ = \frac{\prod_{k=0}^{\infty} x^2}{8} - \frac{1}{8} x^2 \rightarrow \begin{cases} \text{no!} & \text{yes!} \\ \frac{1}{8} + \left( \frac{1}{8} \right) x^2 \rightarrow \text{real?} \end{cases} \]

Polynomial! Yes! Real?

\[ f(x) = 4 - \frac{5}{x} \rightarrow \frac{4}{1} - \frac{5}{1} \cdot \frac{1}{x} = 4 - 5 \cdot x^{-1} \]

These are all the same

Not a polynomial!!

---

**Theorem:** The product of two or more polynomials is also a polynomial. Its degree is the sum of the degrees of the polynomials being multiplied.

\[ 4 + x^{100} + x^2 \text{ (degree 100)} \]

\[ 4 + x + 100x^2 \text{ (degree 3)} \]
§ 5.1 cont

(1) Polynomial? \( G(x) = 8(x-4)^3(x^2+1) \) 
   \( (x-4)(x-4) \) polynomial, degree of 1
   \( (x^2+1) \) polynomial, degree 2
   \( \text{Total, degree 4} \) → it is a polynomial

What's the degree of \( f(x) = 6x + x^{-1} \)
Not a polynomial; blc of neg. exponent

Roots, Zeros, and x-Intercepts.
(These all mean the same thing.)
(All roots/zeros are x-intercepts.)
Replace \( y \) with zero and solve to obtain x-intercepts/zeros/roots.

\[ 3 \rightarrow \square \rightarrow 0 \quad 3 \text{ is a "zero" of } f \]
\[ 5 \rightarrow \square \rightarrow 0 \quad 3 \text{ is on x-int of } f \]
\[ 11 \rightarrow \square \rightarrow 0 \quad 3 \text{ is a root of } f \]
§ 5.1 cont

\[ 0 = 0.004 \times (x-3)(x+6)^3 \]
\[ 0.004 \times (x-3)(x+6)^3 = 0 \]
\[ 0.004 \times (x-3)(x+6)(x+6)(x+6) = 0 \]

Using the zero product property —
\[ x = 0 \] (this answer appears twice)
\[ x = 3 \] (appears once)
\[ x = -6 \] (appears three times)

Therefore, the solutions, 0, 3, and -6, are the real roots/zeroes of the equation.

**Multiplicity**

The number of times a particular root/zero/\( x \)-int appears in an answer is called the multiplicity of that root.

Multiplicities and roots go hand in hand —

**Touching or crossing?**

If a root/zero is of **even** multiplicity, the graph \[ \text{TOUCHES} \] the \( x \)-axis.

§ 5.1 cont.

Multiplicity cont.

If a root/zero of \( f(x) \) is of \( \text{ODD} \) multiplicity, the graph \( \text{CROSSES} \) the \( x \)-axis there.

Graph \( y = 0.004x^2 (x-3)(x+6)^3 \)

Crossing at \( 3 \) (\( x \)) \( \text{Odd crosses!} \)

Crossing at \( \text{even touches!} \)
§ 5.1 cont. pg. 60

(a) List each real zero and its multiplicity.
(b) Determine whether the graph crosses or touches the x-axis.

33. \( f(x) = 5(x^2 + 9)(x-4)^3 \)

\[ y = 5(x^2 + 9)(x-4)^3 \]
\[ 0 = 5(x^2 + 9)(x-4)^3 \]
\[ 0 = 5(x^2 + 9)(x-4)(x-4)(x-4) \]
\[ 5 = 0 \quad x^2 + 9 = 0 \quad \text{set} = 0 \quad \text{set} = 0 \quad \text{set} = 0 \]
\[ x = 4 \quad x = 4 \quad x = 4 \]

False! \( x^2 = -9 \Rightarrow x^2 = \pm \sqrt{-9} \) not real,

contradiction, \( x = \pm 3i \) \[ \text{no solution} \quad \text{root} = 4 \]

multiplicty = 3

\( \text{croses the x-axis, because the multiplicity is odd} \)
\[ f(x) = -7x^3(x^2-7) \]
\[ y = -7x^3(x^2-7) \]
\[ 0 = -7 \cdot x \cdot x \cdot (x^2-7) \]
\[ -7 = 0, \quad x = 0 \quad x = 0, \quad x^2 - 7 = 0 \]
\[ \text{No root } x = 0 \text{ has multiplicity} \]
\[ x^2 = 7 \]
\[ x = \pm \sqrt{7} \]

These are two different numbers!!

\[ \{ 0, \sqrt{7}, -\sqrt{7} \} \]

Multiplicity = 4

-touches crosses
-touches crosses