7.6 Radical equations and problem solving
Steps to Solving a Radical Equation

4. Solve.
\[ \sqrt{x+2} = \sqrt{x-25} = 9 \]

Step 1. Isolate one radical on one side of the equation. (It is alone on one side of the equation with nothing outside of the radical.)
\[ \sqrt{x+2} = 9 - \sqrt{x-25} \]
\[ \frac{\sqrt{x+2}}{\sqrt{x-25}} = 9 - \sqrt{x-25} \]

Step 2. Raise each side to a power that matches the index of the isolated radical.
\[ (\sqrt{x+2})^2 = (9 - \sqrt{x-25})^2 \]
\[ x + 2 = 81 - 18 \sqrt{x+2} + (\sqrt{x-25})^2 \]
\[ x - 25 = 81 - 18 \sqrt{x+2} + x + 2 \]

We went from 2 radicals to 1.

Step 3. If the equation still has a radical, repeat steps 1 and 2.
\[ x - 25 = 83 + x - 18 \sqrt{x+2} \]
\[ x - 108 = x - 18 \sqrt{x+2} \]
\[ -108 = -18 \sqrt{x+2} \]
\[ -6 = \sqrt{x+2} \]

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7.6 Radical equations and problem solving

Steps to Solving a Radical Equation

1. **Solve**
   
   \[
   \sqrt{x+2} + \sqrt{x-25} = 9
   \]

   **Step 1.** Isolate one radical on one side of the equation (It is alone on one side of the equation with nothing outside of the radical.)
   
   \[
   \sqrt{x+2} + \sqrt{x-25} \cdot \sqrt{x+2} = 9 - \sqrt{x+2}
   \]

   \[
   \sqrt{x+2} = 9 - \sqrt{x+2}
   \]

   **Step 2.** Raise each side to a power that matches the index of the isolated radical.
   
   \[
   \left(\sqrt{x-25}\right)^2 = \left(9 - \sqrt{x+2}\right)^2
   \]

   \[
   x - 25 = 81 - 9\sqrt{x+2} - 9\sqrt{x+2} + \sqrt{x+2} \cdot \sqrt{x+2}
   \]

   \[
   x - 25 = 81 - 18\sqrt{x+2} + (\sqrt{x+2})^2
   \]

   \[
   x - 25 = 81 - 18\sqrt{x+2} + x + 2
   \]

   We went from 2 radicals to 1.

   **Step 3.** If the equation still has a radical, repeat steps 1 and 2.

   \[
   \sqrt{x-25} = \frac{83}{\sqrt{83}} + x - 18\sqrt{x+2}
   \]

   \[
   \frac{x - 108}{-x} = x - 18\sqrt{x+2}
   \]

   \[
   \frac{-108}{-x} = -18\sqrt{x+2}
   \]

   \[
   -18 = -18\sqrt{x+2}
   \]

   \[
   6 = \sqrt{x+2}
   \]

   **Cont on next pg**
\[ \frac{6}{3} \sqrt{x + 2} - \frac{3}{2} = x + 2 \]
\[ \frac{1}{3} \sqrt{x + 2} = x \]

Step 4: Check answer
Plug \( x = \frac{79}{3} \) into original problem
\[ \sqrt{x + 2} + \sqrt{x - 25} = 9 \]
\[ \sqrt{\frac{79}{3} + 2} + \sqrt{\frac{79}{3} - 25} = 9 \]
\[ \frac{9}{3} + 3 = 9 \]
\[ 3 = 9 \checkmark \]

\( x = \frac{79}{3} \)
14. Solve

\[ x - \sqrt{6 - 5x} = -12 \]

1. Isolate

\[ x = -12 + \sqrt{6 - 5x} \]

\[ +12 \quad +12 \]

\[ x + 12 = \sqrt{6 - 5x} \]

2. Raise

\[(x + 12)^2 = (\sqrt{6 - 5x})^2 \]

\[ (x + 12)(x + 12) = 6 - 5x \]

Goal: \( x = 0 \) or \( x = 1 \)

3. No repeat

\[ x^2 + 24x + 144 = 6 - 5x \]

\[ +5x \quad -6 \quad -6 + 5x \]

\[ x^2 + 29x + 138 = 0 \]

\[ (x + 6)(x + 23) = 0 \]

Use Zero Product Property

\[ x + 6 = 0 \quad \text{or} \quad x + 23 = 0 \]

\[ -6 \quad -23 \]

\[ x = -6 \quad \text{or} \quad x = -23 \]

14. Check

\[ x = -6 \]

\[ x - \sqrt{6 - 5(-6)} = -12 \]

\[ -6 - \sqrt{6 - 5(-6)} = -12 \]

\[ -6 - \sqrt{6 + 30} = -12 \]

\[ -6 - \sqrt{36} = -12 \]

\[ -6 - 6 = -12 \]

\[ -12 = -12 \quad \checkmark \]

\[ x = -6 \]

\[ x = -23 \]

\[ -23 - \sqrt{6 - 5(-23)} = -12 \]

\[ -23 - \sqrt{6 + 115} = -12 \]

\[ -23 - 121 = -12 \]

\[ -23 - 11 = -12 \]

\[ -23 - 11 = -12 \]

\[ -34 = -12 \]

False