§ 6.8 Exponential Growth & Decay Models

(lecture notes p.4/8)

Method 1: Short way

Method 2: Long way \[ A(t) = A_0 e^{kt} \]

A of t equals A naught "e" to the "kt"

3. The population of a mosquito colony obeys the law of uninhibited growth.

<table>
<thead>
<tr>
<th>Amount of time (d)</th>
<th>Number of Mosquitoes</th>
<th>( A_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 days</td>
<td>1000 mosquitoes</td>
<td>( A_0 )</td>
</tr>
<tr>
<td>2 days</td>
<td>1900 mosquitoes</td>
<td>( A_0 )</td>
</tr>
<tr>
<td>4 days</td>
<td>3610 mosquitoes</td>
<td>( A_0 )</td>
</tr>
<tr>
<td>6 days</td>
<td>?</td>
<td>( A_0 )</td>
</tr>
</tbody>
</table>

What is the size of the colony after 6 days?

How long will it take to have a population of 70,000?

6 day \( \rightarrow 6559 \) mosquitoes

\[ A(t) = A_0 e^{kt} \]

\( A(2) = 1000 e^{2k} \)

plug in any data point other than the first point
§ 6.8 Exponential Growth & Decay Models

(Metric Notes p. 94 e)

Method 1: short way

Method 2: long way \( A(t) = A_0 e^{kt} \)

A of + e = \( e^0 \) naught "e" to the "k" power

(3) The population colony of mosquitoes obey the law of uninhibited growth.

<table>
<thead>
<tr>
<th>Amount of time ( t )</th>
<th>Number of Mosquitoes ( N(t) )</th>
<th>( A_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 days</td>
<td>1000 mosquitos multiply by 1.9</td>
<td>1000</td>
</tr>
<tr>
<td>2 days</td>
<td>1900 mosquitos</td>
<td>1000</td>
</tr>
<tr>
<td>4 days</td>
<td>3610 mosquitos</td>
<td>1000</td>
</tr>
<tr>
<td>6 days</td>
<td>?</td>
<td>1000</td>
</tr>
</tbody>
</table>

What is the size of the colony after 6 days?

How long will it take to have a population of 70,000?

6 days → 6859 mosquitos

8 days → 13032.1
10 day → 24760.99
12 day → 47035.32
14 day → 87387.17

\( A(t) = A_0 e^{kt} \)

\( A(0) = 1000 \)

\( A(t) = 1000 e^{kt} \)

plug in any data point other than the first point.

70,000 does not appear on the table so method 1 does not work.
\[ A(t) = 1000e^{kt} \]
\[ \frac{0.859}{1000} = e^{\frac{k}{1000}} \]
\[ 0.859 = e^{\frac{k}{1000}} \]
\[ \ln 0.859 = \frac{k}{1000} \]
\[ k = \frac{\ln 0.859}{600} \]

Now use \( A_0, e, \) & \( k \):
\[ \frac{A(t)}{A_0} = e^{\frac{kt}{A_0}} \]
\[ \frac{A(t)}{1000} = e^{\frac{kt}{1000}} \]
\[ \ln 0.859 = \frac{kt}{1000} \]
\[ \ln 70 = \frac{\ln 0.859}{600} \]
\[ \ln 70 = \frac{\ln 0.859}{600} \]
\[ \ln (70) / (\ln 0.859 / 600) = 2 \]
\[ 70 = e^{2 \cdot 0.859/600} \]

13.2 days
(2) Strontium 90 is radioactive material that decays according to the function \( A(t) = A_0 e^{-0.0244t} \), where \( A_0 \) is the initial amount present & \( A \) is the amount present a time \( t \) (in years). Assume that a scientist has a sample of 800 grams.

\[ A(t) = A_0 e^{-0.0244t} \]

(a) What is the decay rate of Strontium-90? 

\[ A_0 = 800 \text{ grams} \]

\( k = -0.0244 \) \( \Rightarrow \) has to be converted to \( \% \)

\( k = 2.44 \) \( \text{(decay)} \) \( \text{answer} \)

(b) How much Strontium-90 is left after 20 years?

\[ A(t) = A_0 e^{-0.0244t} \]

Plug in "t" = 20

\[ A(20) = 800 e^{-0.0244 \times 20} \]

= 491.0829 grams

(c) What is the half-life of Strontium-90?

Half-life is the length of time needed for a substance to decay to half of its original amount.

\[ A(t) = 800 e^{-0.0244t} \]

This is half of that

\[ 400 = 800 e^{-0.0244 \times 20} \]

\[ \frac{400}{800} = e^{-0.0244 \times 20} \]

\[ \frac{1}{2} = e^{-0.0244 \times 20} \]

\[ \ln \frac{1}{2} = -0.0244 \times 20 \]

\[ -0.0244 \times 20 = \ln \frac{1}{2} \]

\[ \frac{28.4}{0.244} \approx t \]
The half-life of carbon-14 is 5600 years. If charred logs from an old log cabin show only 64% of the carbon-14 expected in living matter, when did the cabin burn down? Assume that the cabin burned soon after it was built from freshly cut logs.

\[ A(t) = A_0 e^{kt} \]

The "k" value can be discovered from the half-life.

\[ \frac{1}{2} A_0 = A_0 e^{k \times 5600} \]

After 5600 years, the amount \( A_0 \) becomes \( \frac{1}{2} A_0 \).

\[ \frac{1}{2} = e^{k \times 5600} \]

Now plug in \( A_0 \) (sort of) and \( e^{k} \).

\[ \ln \frac{1}{2} = k \frac{5600}{5600} \]

\[ \ln \frac{1}{2} = k \]

Now use amount 64% to get "t".

\[ \ln 0.64 = \frac{\ln \frac{1}{2} \times t}{5600} \]

\[ \ln 0.64 = \frac{\ln \frac{1}{2}}{5600} \cdot t \]

\[ 2\ln 0.64 = 7 \]

\[ 3605 \cdot 0.64 = 7 \]