Functions

Relation - A correspondence between two sets. The first set is the domain, the second set is the range. If \( x \) belongs to the domain and \( y \) belongs to the range, and there is a relation between \( x \) and \( y \), we say that \( y \) depends on \( x \), or \( x \) corresponds to \( y \).

Representing relations
- equations, graphs, mapping, ordered pairs.

Function - A relation that associates each element \( x \) of the domain with exactly one element \( y \) of the range.

Domain - The set of all inputs \( x \), independent variable, the argument.
Range - The set of all outputs \( y \), dependent variable, the image of \( x \).

Function = Predictable correspondence; each given input will produce a unique output.

A relation is a function if:
- The first element in the set of ordered pairs does not repeat.
- There are no multiple arrows leaving the same input.
- Solving the equation for \( y \) will only return one solution.
- The vertical line test: A vertical line will intersect the graph in no more than one point.
Function Notation

We rewrite the equation using a symbolic notation $f(\, x\,)$.
$y = f(x)$ will read "f of x" or "f at x".
$f(x)$ is the value of f at the number $x$. 

Practice

Determine whether the relations below are functions:

1. $R = \{(1,3), (2,5), (-1,5), (1,7)\}$

2. Make a table for the given values:

<table>
<thead>
<tr>
<th>Car</th>
<th>MPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honda Accord</td>
<td>31</td>
</tr>
<tr>
<td>Toyota Camry</td>
<td>29</td>
</tr>
<tr>
<td>Kia Optima</td>
<td>35</td>
</tr>
</tbody>
</table>

3. Graph the function:

\[ y = \_x \]
Function Values

Ex. 1  Let \( f(x) = 4x^2 - 5 \). Evaluate the following:

\[
\begin{align*}
\hat{f}(3) &= 7 \\
\hat{f}(-4) &= 7 \\
\hat{f}(\frac{1}{2}) &= 7
\end{align*}
\]

Solution:

\[
\begin{align*}
\hat{f}(3) &= 4 \cdot 3^2 - 5 = 36 - 5 = 31 \\
\hat{f}(-4) &= 4 \cdot (-4)^2 - 5 = 64 - 5 = 59 \\
\hat{f}(\frac{1}{2}) &= 4 \cdot \left(\frac{1}{2}\right)^2 - 5 = 4 \cdot \frac{1}{4} - 5 \\
&= 1 - 5 = -4
\end{align*}
\]

Ex. 2  Let \( f(x) = \frac{3}{x-5} \).

Evaluate \( f(2) \), \( f(0) \), and \( f(5) \).

\[
\begin{align*}
\hat{f}(2) &= \frac{3}{2-5} = \frac{3}{-3} = -1 \\
\hat{f}(0) &= \frac{3}{0-5} = -\frac{3}{5} \\
\hat{f}(5) &= \frac{3}{5-5} = \frac{3}{0} \text{ undefined}
\end{align*}
\]