Section 7.4 Consumer/Producer Surplus

Consumer surplus = money they were willing to spend but saved b/c able to get for cheaper.

Producer surplus = change they were willing to sell item for, but they made more than bottom line.

Formula: Consumer Surplus

\[ CS = \int_0^\infty D(q) dq - \bar{p} \bar{q} = \int_0^\infty (\bar{q} - \bar{p}) dq \]

\(\bar{p}, \bar{q}\) = selling price & selling quantity

\(p = D(q) = \text{Demand} = \ldots \ldots \quad q\)

*looking for \(p = \ldots \ldots \quad q\)

Formula: Producer Surplus

\[ PS = \int_0^\infty \left( \bar{p} - S(q) \right) dq \]

\(S(q) = \text{Supply}\)

\(p = S(q) = \ldots \ldots \quad q\)

*looking for \(p = \ldots \ldots \quad q\)
Ex: 1. pg 546  \( p = 15e^{-0.01q} \) Find \( CS(\text{consumer surplus}) \) when you sell \( CP's \) for \$5.00.

**Step 1**  
\( \bar{p} = $5.00 \)

**Step 2**  
so  \( \frac{5}{15} = \frac{15e^{-0.01q}}{15} \)

**Step 3**  
\( \frac{5}{15} = e^{-0.01\bar{q}} \)  
\( j \ln \frac{1}{3} = \ln e^{-0.01\bar{q}} \)

**Step 4**  
\( \ln(\frac{1}{3}) = \frac{-0.01\bar{q}}{-0.01} \)  
\( j \frac{\ln(\frac{1}{3})}{-0.01} = \bar{q} \)

**Step 5**  
109.86 = \bar{q}

so  \( \bar{p} = $5 \)  
\( \bar{q} = 109.86 \)  
\( \bar{D}(q) = 15e^{-0.01q} \)

**Step 6** Put #1's into formula

\[
CS = \int_{\emptyset}^{\bar{q}} D(q) dq - \bar{p} \bar{q} = \int_{\emptyset}^{109.86} 15e^{-0.01q} dq - (5e^{109.86})
\]

\[
= 15 \left[ -\frac{1}{0.01} e^{-0.01q} \right]_{0}^{109.86} - 549.306
\]

\[
= 15 \left[ -\frac{1}{0.01} e^{-0.01 \times 109.86} \right] - 549.306
\]

\[
= 15 \times \left[ -\frac{1}{0.01} e^{-0.01 \times 109.86} \right] - 549.306
\]

\[
= 999.99 - 549.306 = $450.69 = CS
\]
Proper Surplus

Ex: #2 pg 547

Need to have a \( p \) and \( q \) \( p = S(q) = \ldots q \)

**Step 1**
\( q = 20\sqrt{p-4} \) "Wrong Form - need" \( p = \ldots q \)

B/c this is \( (q) \) she is willing to provide.

* Solve for \( p \) to get \( p = \ldots q \) format.

\[
\frac{a}{20} = \sqrt{p-4} = \frac{a^2}{400} = p - 4 + 4
\]

So \( \frac{a^2 + 4}{400} = p \) * Proper format

**Step 2** We were given \( \bar{p} = \$8 \), now solve for \( \bar{q} \) by plugging in \( p \) value to original equation.

\[
\frac{a^2}{400} = 20\sqrt{8-4} = 20\sqrt{4} = 40
\]

**Step 3** \( p = \$8 \) \( \bar{q} = 40 \) \( S(q) = \frac{a^2 + 4}{400} \)

Plug into formula: \( PS = \int_0^{\bar{q}} (p - S(q)) \, dq \)

\[
= \int_0^{40} 8 - \left( \frac{a^2}{400} + 4 \right) \, dq = \int_0^{40} (8 - \frac{a^2}{400} + 4) \, dq
\]

\[
= \int_0^{40} \left(4 - \frac{a^2}{400}\right) \, dq \quad \text{* Now integrate*}
\]
Step 4: \[ \int_0^{40} \left( 1 - \frac{q^2}{400} \right) dq = \left. \frac{4q - \frac{1}{400} q^3}{3} \right|_0^{40} \]

Step 5: Plug into calculation to solve

2nd Trace \[ y = 4 - \frac{(q^2)}{400} \]

2nd Trace \# 7

Lower limit = 0 \quad Upper limit = 40

Enter: \[ \$106.67 \] Producer Surplus

Ex 3: \# 3  \quad Equilibrium

Step 4: Find equilibrium number

Supply = demand

\[ S(q) = D(q) \]

\[ p = \frac{q^2}{400} + 4 \quad q = \sqrt{200(16-p)} \]

\[ q = 20\sqrt{p-4} \]

So easier to use this
Step 2: \[20 \sqrt{p-4} = \sqrt{200(16-p)}\]

Square both sides to get rid of radicals, so

\[400(p-4) = 200(16-p)\]

\[400p - 1600 = 3200 - 200p\]

Step 3: Clean up algebra and get \(p\) by itself

\[600p = 4800\]

\[p = \frac{4800}{600} = 8\]

Step 4: Change to meet equilibrium

So \(20 \sqrt{8-4} = 20 \sqrt{4} = 40\)

Step 5: So you need to sell 40 shirts @ \$8 each

Step 6: \[CS = \bar{p} = \frac{P}{Q} = \$8\]

Step 6: \[\frac{P}{Q} = \$40\]

Step 6: \[\bar{q} = \cdots\]

So now take \(q = \sqrt{200(16-p)}\) * solve for \(p\)

\[q^2 = 200(16-p)\]

\[\frac{q^2}{200} = 16-p\]

\[\frac{q^2}{200} = 16-p\]

\[\frac{q^2}{200} - 16 = -p\]

\[16 - \frac{q^2}{200} = p\]
Step 7: now we have 
\[ p = 8 \]
\[ q = 40 \]
\[ D(q) = 16 - \frac{q^2}{200} \]

Step 8: Plug into formula:
\[ CS = \int_{0}^{40} D(q) dq - \overline{q} \overline{p} \]
\[ = \int_{0}^{40} 16 - \frac{q^2}{200} \, dq - (8)(40) \]

Step 9: Plug into calculator
\[ y = \ldots \]
\[ 2^\text{nd Calc} \# 7 \]
\[ \text{Lower limit} = 0 \]
\[ \text{Upper limit} = 40 \]
\[ \text{Enter} = 533.33 \]
\[ = 533.33 - (8 \times 40) = 533.33 - 320 = 213.33 \]
\[ = CS \]

\[ \int_{0}^{40} 213.33 + 106.67 = \int_{0}^{40} \text{Total Social Gain} = 320,000 \text{ from Ex#3} \]

\[ \text{From Ex#2} \]
Continuous Income Streams