Difference between a Left & a Right Riemann Sum

Formula to use & remember:

Left Riemann Sum: \[ \sum_{k=0}^{n-1} f(x_k) \Delta x = h \cdot B \]

i.e., Above pts. (1-5) are \( k=0 \) is begin @ 1
\( n-1 \) is \# 5 less 1, so stop @ 4.
Right Riemann Sum \[ \sum_{k=1}^{n} f(x_k) \Delta x \]

\[ f(x) = x^2 \quad n = 4 \quad [1, 5] \]

\[ \Delta x = \frac{5-1}{4} = 1 \]

**Left Riemann Sum**

\[ f(1) = 1^2 = 1 \]
\[ f(2) = 2^2 = 4 \]
\[ f(3) = 3^2 = 9 \]
\[ f(4) = 4^2 = 16 \]

Now, find \[ \sum_{k=0}^{n-1} f(x_k) \Delta x \]

\[ \sum_{k=0}^{3} f(x_k) \Delta x = \frac{3}{4} \sum_{k=0}^{3} f(x_k) \Delta x \]

\[ f(1) \Delta x + f(2) \Delta x + f(3) \Delta x + f(4) \Delta x \]

\[ = (1)^2(1) + (2)^2(1) + (3)^2(1) + (4)^2(1) = 1 + 4 + 9 + 16 = 30 \]

If it had been a **Right Riemann Sum**

Traditional way

\[ \int f(x) \Delta x \]

\[ = f(1) \Delta x + f(2) \Delta x + f(3) \Delta x + f(4) \Delta x \]

\[ = (1)^2(1) + (2)^2(1) + (3)^2(1) + (4)^2(1) = 1 + 4 + 9 + 16 = 30 \]

So, how to get the \# to be more accurate estimate

We take \[ \frac{30 + 54}{2} \]

\[ = \frac{84}{2} = 42 \]

Better/closer average approx.
What we are actually doing is

\[ \int_{1}^{5} x^2 \, dx = \left. \frac{x^3}{3} \right|_{1}^{5} = \frac{5^3 - 1^3}{3} = \frac{125 - 1}{3} = \frac{124}{3} \]

\[ = 41 \frac{1}{3} \text{ closer to average of } \frac{41}{2}. \]

From pg 485

\[ \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{k=1}^{n-1} f(x_k) \Delta x \]

Ex: \[ f(x) = x^3 \quad n = 6 \quad \left[ 1, 5 \right] \]

\[ \Delta x = \frac{b-a}{n} = \frac{5-1}{6} = \frac{2}{3} \]

\[ \begin{array}{c}
\text{Left Riemann Sum} \\
\hline
n=6 \\
1. f(1) = 1^3 \\
2. f(1^{2/3}) = 2^{2/3} \\
3. f(4^{1/3}) = 3^{1/3} \\
4. f(3^{2/3}) = 4^{1/3} \\
5. f(3^{2/3}) = 5 \\
\end{array} \]

Total \cdot \Delta x \text{ gives approximation}

\[ \begin{array}{c}
\text{Right Riemann Sum} \\
\hline
\text{w/ left} \\
1. f(1) = 1^3 \\
2. f(1^{2/3}) = 2^{2/3} \\
3. f(4^{1/3}) = 3^{1/3} \\
4. f(3^{2/3}) = 4^{1/3} \\
5. f(3^{2/3}) = 5 \\
\end{array} \]

\[ \text{w/ right} \]

\[ \begin{array}{c}
f(1^{2/3}) + f(4^{1/3}) + f(2^{2/3}) + f(3) + f(3^{2/3}) + f(5)
\end{array} \]

Total \cdot \Delta x = \text{Width}