Part 2 4.6 Section: Logarithmic Differentiation

Uses properties of logarithms.
I.e. \( \ln(xy) = \ln(x) + \ln(y) \)
\[ \ln \frac{x}{y} = \ln(x) - \ln(y) \]
\[ \ln x^4 = 4 \cdot \ln x \]

3 Rules that help make derivatives easier to find.

\( \#44 \)

\( y = (3x+2)(8x-5) \)

**Step 1** Take log of both sides, so

\( \ln y = \ln((3x+2)(8x-5)) \)

**Step 2** Use rules above to get addition problem, so \( \ln\text{ its multiplication use } \ln x + \ln y \)

Thus \( \ln y = \ln(3x+2) + \ln(8x-5) \)

**Step 3** Find derivatives

\[ \frac{1}{y} \cdot y' = \frac{1}{3x+2} \cdot 3 + \frac{1}{8x-5} \cdot 8 \]

**Step 4** Simplify:

\[ y' = \left( \frac{3}{3x+2} + \frac{8}{8x-5} \right) y \]

With logarithmic this would be final answer.

Re-do by plugging in \( y' \) into equation.

so

\[ y' = \left( \frac{3}{3x+2} + \frac{8}{8x-5} \right)(3x+2)(8x-5) \]
\[ = 3(8x-5) + 8(3x+2) \]
\[ = 24x - 15 + 24x + 16 \]
\[ = 48x + 1 \]
Algebraically working it out:

\[ y = (3x + 2)(8x - 5) \]

\[ \text{FOIL} \quad y = 24x^2 - 15x + 16x - 10 \]

\[ y = 24x^2 + x - 10 \]

Now find \( y' \) by simplifying:

\[ y' = 48x + 1 \]

*Use this to recheck/verify previous answer.

**Example:**

\[ y = \frac{(2x - 1)^{18} (6x - 3)^{12} (5x + 1)^3}{(2x - 3)^{14} (4x + 3)^6} \]

**Step 1:**

External power - Get rid of it by multiplying it by internal powers, so

\[ y = \frac{(2x - 1)^{36} (6x - 3)^{24} (5x + 1)^6}{(2x - 3)^{34} (4x + 3)^{12}} \]

**Step 2:**

Get powers down by using \( \ln \), so

\[ \ln y = \ln \left( \frac{(2x - 1)^{36} (6x - 3)^{24} (5x + 1)^6}{(2x - 3)^{34} (4x + 3)^{12}} \right) \]

**Step 3:**

Use \( \ln x + \ln y + \ln x - \ln y \) rules, so

\[ \ln y = 36 \ln (2x - 1) + 24 \ln (6x - 3) + 6 \ln (5x + 1) - 34 \ln (2x - 3) - 12 \ln (4x + 3) \]

**Step 4:**

Take derivative of each 5 pieces individually

\[ \frac{1}{y} \cdot y' = 36 \cdot \frac{1}{2x - 1} + 24 \cdot \frac{6}{6x - 3} + 6 \cdot \frac{5}{5x + 1} - 34 \cdot \frac{2}{2x - 3} - 12 \cdot \frac{4}{4x + 3} \]

**Simplified:**

\[ y' = \left( \frac{36}{2x - 1} + \frac{144}{6x - 3} + \frac{30}{5x + 1} \right) \cdot \frac{1}{y} \]

**Step 5:**

Multiply by \( y' \) to get

\[ y^2 \]

So

\[ y^2 = \left( \frac{72}{2x - 1} + \frac{144}{6x - 3} + \frac{30}{5x + 1} \right) \cdot \left( \frac{144}{6x - 3} + \frac{30}{5x + 1} \right) \cdot \frac{1}{y} \]