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March 28 - Section 6.5 - Properties of Logarithms

- Definition of a logarithm: Also called a "logarithmic formula"
  - \( \log_b x = y \Rightarrow b^y = x \)
  - Transforms from a log... into an exponent

- \( \log_a 1 = 0 \) - The log base \( a \) of 1 is always 0; \( a^0 = 1 \)

- \( \log_a a = 1 \) - The log base \( a \) of itself is 1; \( a^1 = a \)

- \( \log_a (mn) = \log_a m + \log_a n \) - The log of a product is the sum of logs of the factors

- \( \log_a \left( \frac{m}{n} \right) = \log_a m - \log_a n \) - The log of a quotient is the difference of logs of the factors

- \( \log_a b = \frac{\log_b c}{\log_c a} \) - Change of base formula; this is useful to transform a log
- \( \log_a a = 1 \) - In a more useful form

- \( \log_a a^m = m \cdot \log_a a \)

- If \( m = n \), then \( \log_b m = \log_b n \) - This says we can apply logs to an equation by applying them to both sides

- \( \log_a \log_m = \frac{\log_b \log_m}{\log_b a} \) - Logs and exponents are inverses. If you have a box 'a', with a log, their inverse acts as a cancellation, and you are left with your argument, 'm', the having you are talking to log off.
14. **homework:** \( \log_2 2^{-13} = -13 \cdot \log_2 2 = -13 \cdot 1 = -13 \)

16) \( \ln e^{\sqrt{2}} \Rightarrow \log e^{\sqrt{2}} = \sqrt{2} \cdot \log e = \sqrt{2} \)

\[ \text{Invert cancellation!} \]

18) \( e^{\ln 8} = e^{\log 8} = 18 \)

\[ \Rightarrow 2 \log e^{8} = \log e^{8^2} = 8^2 = 64 \]

\[ \star e^{\log 16} = \sqrt{16} \]

\[ \star e^{\log e^4} = 4^2 = 16 \]

20) \( \log_b 9 + \log_b 4 = \log_b (36) = 2 \cdot 6^2 = 36 \)

\[ \Rightarrow \text{Product rule} \]

22) \( \log_8 16 - \log_8 2 = \log_8 \left( \frac{16}{2} \right) = \log_8 8 = 1 \]

24) \( \log_3 8 \cdot \log_9 9 = \frac{\log_3 8}{\log_3 9} \cdot \frac{\log_9 9}{\log_9 9} = \frac{\log 8}{\log 9} \cdot \frac{\log 9}{\log 9} = \frac{\log 8}{\log 9} \)

\[ \Rightarrow 2 \frac{\log 3}{\log 3} = \square \]

26) \( 5 \log_3 1 + \log_3 7 = \square 5 \log_3 (42) = 14 \)

\[ \Rightarrow 5 \log_3 6 + (3 \log_3 7) = 6 (7) = \]

28) \( e^{\log e^3} = e^3 = 3 \)

30) \( \ln 2 = \alpha, \ln 3 = \beta \)

\[ \Rightarrow \ln 2 - \ln 3 = \alpha - \beta \]
2) \( \ln(0.5) = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2 \)

**Expand/Contract Logs**

\[
\log\left(\frac{x^2 \cdot y^{1/2}}{z^{3/4} \cdot w^5}\right) = \log(x^2) + \log(y^{1/2}) - \log(z^{3/4}) - \log(w^5)
\]

\[
= 2 \log x + \frac{1}{2} \log y - \frac{3}{4} \log z - 5 \log w
\]

\[
\log_2\left(\frac{\sqrt{x} \cdot \sqrt[3]{y}}{w^{3/8} \cdot z^{5/8}}\right) = \log_2(\sqrt{x}) + \log_2(\sqrt[3]{y}) + \log_2(z^{1/8}) - \log_2(w^{3/8}) - \log_2(z^{5/8})
\]

\[
= \frac{1}{2} \log_2 x + \frac{1}{3} \log_2 y + \frac{1}{8} \log_2 z - \frac{3}{8} \log_2 w - \frac{5}{8} \log_2(z^{5/8})
\]

With these we can then bring down as a coefficient

\(\text{Rewrite as a single logarithm}\)

\[
\ln x - 6 \ln y + \frac{1}{2} \ln z - \frac{3}{4} \ln k
\]

\[
\ln y^4 - \ln y^6 + \frac{1}{2} \ln z^2 - \ln z^{5/2}
\]

\[
\ln\left(\frac{x^{4/3} \sqrt{z}}{y^2 \sqrt[3]{k}}\right)
\]

\(\text{Section 6.3: Exponential equations with like bases}\)
\[ 76) \quad 9^{2x} \cdot 27^{x^2} = 3^{-1} \]

\[ (3^2)^{2x} \cdot (3^3)^{x^2} = 3^{-1} \]

\[ (3)^{4x} \cdot (3)^{5x^2} = (3)^{-1} \]

\[ (3)^{3x^2 + 4x} = (3)^{-1} \]

\[ 4x + 3x^2 = -1 \]

\[ 3x^2 + 4x + 1 = 0 \]

\[ (3x + 1)(x + 1) = 3x - 1 \Rightarrow x = -\frac{1}{3} \]

\[ 3y = 1 \Rightarrow y = \frac{1}{3} \]

\[ 2^{3x} = 3^{3x - 1} \]

- Equations w/ unlike bases

× remember \( m = n \) ⇒ \( \log m = \log n \)

\[ \ln 2^{3x} = \ln 3^{2x - 1} \Rightarrow (3x) \ln 2 = (2x - 1) \ln 3 \]

\[ \Rightarrow x(3 \ln 2) = x(2 \ln 3) - \ln 3 \]

\[ \Rightarrow x(\ln 2 - \ln 3) = -\ln 3 \]

\[ \Rightarrow x(\ln 2 - \ln 3) = -\ln 3 \]

\[ \Rightarrow x = \frac{-\ln 3}{\ln \left(\frac{2}{3}\right)} \]

\[ \Rightarrow (\text{decimal}) = (2x - 1)(\text{decimal}) \]

\[ 3x (0.6931) = (2x - 1)(1.0986) \]