3.5 Equations of Lines

Point-slope form
\[ y - y_1 = m(x - x_1) \]

or
\[ m = \frac{y - y_1}{x - x_1} \]

Slope-intercept form
\[ y = mx + b \]

(solve for \( y \))

Standard form
\[ ax + by = c \]

\( a \) cannot be a fraction
(get all \( x \) and \( y \) on same side)

Ex 1
Write the equation of a line through \((-4, 5)\) and \((5, 3)\). Write your answer in standard form.

goal:
point \((-4, 5)\)
slope
\[ \frac{5 - 3}{-4 - 5} = \frac{-2}{-9} = \frac{2}{9} \]
goal form: \( ax + by = c \)

\[ \frac{y - 5}{x + 4} = \frac{2}{9} \]

\[ 2x + 9y = -18 + 45 \]

\[ 2x + 9y = 27 \]
Proportion version

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ \left( x, y_1 \right), \left( -4, 5 \right) \]
\[ m = -\frac{2}{3} \]

\[ -\frac{2}{3} = \frac{y - 5}{x - (-4)} \]
\[ -\frac{2}{3} = \frac{y - 5}{x + 4} \]

\[ 2(x + 4) = 9(y - 5) \]
\[ 2x + 8 = 9y - 45 \]
\[ 2x + 9y = 53 \]
\[ 2x + 9y = 37 \]

\[ \frac{2}{3} \times \frac{2}{3} \]
\[ \frac{4}{9} \]

\[ \text{ad} = bc \]

\[ \text{Ex.2: } \text{Write the equation of a line through } (-5, 8) \text{ that is perpendicular to the line } 3x - 5y = 7. \text{ Write your answer in slope-intercept form} \]

\[ \text{Point: } (-5, 8) \]

**Slope:**
\[ 3x - 5y = 7 \]
\[ 5y = -3x + 7 \]
\[ y = \frac{3}{5} x + \frac{7}{5} \]
\[ y = \frac{3}{5} \left( x - \frac{7}{5} \right) \]

\[ \text{Need: } -\frac{5}{3} \]

**Proportion**
\[ n = \frac{y - y_1}{x - x_1} \]
\[ -\frac{2}{3} = \frac{y - 5}{x - (-5)} \]
\[ -\frac{2}{3} = \frac{y - 5}{x + 5} \]
\[ \left( y - 8 \right) = -5 \left( x + 5 \right) \]
\[ 3y - 24 = -5x - 25 \]
\[ 3y = -5x - 24 + 24 \]
\[ 3y = -5x - 1 \]
\[ y = -\frac{5}{3} x - \frac{1}{3} \]
8x3  Write the equation of the line through (-5, -3) that is parallel to the line y = 5x + 2. Write your answer in Standard form.

Point: (-5, -3)
Slope: \( y = 5x + 2 \) \parallel \n\text{m} = 5

Need same slope \n\text{m} = 5!

\[
m = \frac{y - y_1}{x - x_1}, \quad \frac{5}{1} = \frac{y - (-3)}{x - (-5)} \Rightarrow \frac{5}{1} = \frac{y + 3}{x + 5}
\]

Goal format: \( ax + by = c \)

\[
5(x + 5) = 1(y + 3) \\
5x + 25 = y + 3 \\
5x - y = 3 - 25 \\
\boxed{5x - y = -22}
\]

Traditional:

\[
y - y_1 = m(x - x_1) \\
y - (-3) = 5(x - (-5)) \\
y + 3 = 5(x + 5) \\
y + 3 = 5x + 25 \\
-5x + y = 25 - 3 \\
\boxed{-5x + y = 22}
\]

\boxed{5x - y = -22}
Ex 4. Write the equation of the vertical line through (-1, 0).

\[ x = -1 \]

\[ m = \text{undefined} \]
\[ x = \text{constant} \]

Line: \[ x = 2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Ex 5. Write the equation of the horizontal line through (-5, -8).

\[ y = -8 \]

Ex 6. Write the equation of the line through (-2, 5) that is parallel to the line \( x = 4 \).

// to \( x = 4 \) (vertical)

\[ \Rightarrow \] vertical line through (-2, 5) \[ x = -2 \]
through \((-4, 6)\) \(\perp\) to \(x = 1\)

horizontal line through \((-4, 6)\) \(y = 6\)

Section 3.7

\[x + 2y > 4\]
dotted.

(1) graph as if it is:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

(2) decide dotted or solid line
\(<\) or \(\leq\) \((<\) or \(\geq\))

(3) decide which part to shade
(test point not on line)

\[(0, 6)\]

\[x + 2y > 4\]
\[0 + 2(6) > 4\]
\[12 > 4\]
\[\text{no}\]
Test 3

grids
\( (h, k) \) a etc.

4 fn questions

sq. root fn. L/R D/R

3 pts space
slow work

plot points
2 - create graphs (line graphs)
intercepts
slope (2 pts and line)
equations of lines - 2
\( \parallel \) \( \perp \) or neither

graph inequality

domain \ range (fns.)
- ordered pairs (no repeat x's
- equation (y')
find \( f(3) \)

formula / graph find \( f(2) \)
D/R graphs

\( \{( x, f(x) \} \)

D: \(( -\infty, \infty )\)
R: \(( -\infty, -1)\)
D: TR \ (\ -\infty,\ \infty\ )
R: TR

graph \ (\ parabola\ )
(square root)

Word problem \ (\ linear\ )
Piece-wise \ find \ f(x) \ identify \ only

\[ f(x) = \text{variable} + \frac{\text{fixed}}{\text{cost}} \]

\[ f(x) = 4x + 2 \]

12 miles
\[ f(12) + 2 = 50 \]

Martin - Gay
2. No worked out answers \ (\ occult\ -\ choice \ section\ )

Lial
Practise all about lines \ - \ solutions
Practise \ \text{fas/parabola/\ family of \ lines - \ solids}
Solve's \ and \ their \ graphs \ - \ 9 \ cases.
\[5x - 2y < 10\]

\[\begin{array}{c|c|c}
  x & y & \text{dotted} \\
  0 & -5 & \\
  2 & 0 & \\
\end{array}\]

test \((0, 0)\)
\[5(0) - 2(0) < 10\]
\[0 < 10\]
yes

test \((1, 1)\)
\[5(1) - 2(1) < 10\]
\[3 < 10\]
no

\[y > 2x\]

\[\begin{array}{c|c|c}
  x & y & \text{no} \\
  0 & 0 & \\
  1 & 2 & \\
\end{array}\]

test \((2, 2)\)
\[0 > 2(2)\]
\[0 > 4\]
no